Group connectivity of graphs with diameter at most 2

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Abstract

Let $G$ be an undirected graph, $A$ be an (additive) abelian group and $A^* = A - \{0\}$. A graph $G$ is $A$-connected if $G$ has an orientation $D(G)$ such that for every function $b : V(G) \rightarrow A$ satisfying $\sum_{v \in V(G)} b(v) = 0$, there is a function $f : E(G) \rightarrow A^*$ such that at each vertex $v \in V(G)$, the amount of $f$ values on the edges directed out from $v$ minus the amount of $f$ values on the edges directed into $v$ equals $b(v)$. In this paper, we investigate, for a 2-edge-connected graph $G$ with diameter at most 2, the group connectivity number $\Lambda_g(G) = \min\{n : G$ is $A$-connected for every abelian group $A$ with $|A| \geq n\}$, and show that any such graph $G$ satisfies $\Lambda_g(G) \leq 6$. Furthermore, we show that if $G$ is such a 2-edge-connected diameter 2 graph, then $\Lambda_g(G) = 6$ if and only if $G$ is the 5-cycle; and when $G$ is not the 5-cycle, then $\Lambda_g(G) = 5$ if and only if $G$ is the Petersen graph or $G$ belongs to two infinite families of well characterized graphs.

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