1. Let $\mathcal{T}$ be a subbasis for a topology $\mathcal{T}$ on $X$.
   (a) Prove that $\mathcal{T} = \bigcap \{ \mathcal{T}' \mid \mathcal{T}' \text{ is a topology on } X \text{ and } \mathcal{T} \subseteq \mathcal{T}' \}$.
   (b) Prove that $\mathcal{T}$ is the smallest topology on $X$ which contains $\mathcal{T}$.

2. Let $(X, d_X)$ and $(Y, d_Y)$ be two metric spaces.
   (a) Prove that each of the following formulas defines a metric on $X \times Y$:
      (i) $d((a_1, a_2), (b_1, b_2)) = \sqrt{(d_X(a_1, b_1))^2 + (d_Y(a_2, b_2))^2}$
      (ii) $\sigma((a_1, a_2), (b_1, b_2)) = d_X(a_1, b_1) + d_Y(a_2, b_2)$
      (iii) $\rho((a_1, a_2), (b_1, b_2)) = \max\{d_X(a_1, b_1), d_Y(a_2, b_2)\}$
   (b) For $X = Y = \mathbb{R}$ and their standard metrics $d_X(a_1, a_2) = |a_1 - a_2|$ and $d_Y(b_1, b_2) = |b_1 - b_2|$, sketch the three $\epsilon$-balls centered at $a = (0, 0)$ of radius $\epsilon = 1$: (i) $B_d(a, \epsilon)$; (ii) $B_\sigma(a, \epsilon)$; (iii) $B_\rho(a, \epsilon)$.

3. Find a linearly ordered set $X$ and a convex subset $Y$ of $X$, such that $Y$ is not any type of interval of $X$.

4. Prove that the order topology on $\mathbb{R} \times \mathbb{R}$ induced by the dictionary order agrees with the product topology $\mathbb{R}_d \times \mathbb{R}$, where $\mathbb{R}_d$ denotes $\mathbb{R}$ in the discrete topology.

5. Let $L$ be a straight line in the Euclidean plane.
   (a) Describe the subspace topology on $L$ when viewed as a subspace of $\mathbb{R}_l \times \mathbb{R}$.
   (b) Describe the subspace topology on $L$ when viewed as a subspace of $\mathbb{R}_l \times \mathbb{R}_l$.
   [Consider cases: (i) $L$ is horizontal, (ii) $L$ is vertical, (iii) $L$ has positive slope, (iv) $L$ has negative slope.]

6. Prove that there is no metric $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ on $\mathbb{R}$ such that its metric topology agrees with the lower limit topology.