1. Consider the knot $5_2$ which is depicted on page 126 of your textbook.
   
   (a) Draw a complete diagram of all states for this knot.
   
   (b) Use your diagram to compute the Kauffman polynomial $F_{5_2}(t)$.

2. (a) Find a resolving tree for the knot $5_2$.
   
   (b) Find a resolving tree for the knot $6_3$.
   
   (c) Use your resolving tree to find $F_{5_2}(t)$ and $P_{5_2}(x, y)$.

3. Let $\bar{K}$ denote the mirror image of the knot $K$. Let $F_K(t)$ and $P_K(x, y)$ denote the Kauffman polynomial and the HOMFLY polynomial of $K$, respectively.
   
   (a) Use the description in terms of states to show that $F_{\bar{K}}(t) = F_K(t^{-1})$.
   
   (b) Use skein relations to prove that $P_{\bar{K}}(x, y) = P_K(x^{-1}, y)$.
   
   (c) Is the knot $5_2$ amphicheiral?

4. Prove that all our knot polynomials are invariants of unoriented knots.

5. This problem indicates the various connections between our polynomials.
   
   (a) Show that $F_L(t)$ can be obtained from $P_L(x, y)$ by substituting $x = t^4\sqrt{-1}$ and $y = (t^2 - t^{-2})\sqrt{-1}$.
   
   (b) Show that $\nabla_L(z)$ can be obtained from $P_L(x, y)$ by substituting $x = \sqrt{-1}$ and $y = z\sqrt{-1}$.
   
   (c) Prove that $V_L(s) = F_L(s^{-1/4})$ and that $\Delta_L(w) = \nabla_L(w^{1/2} - w^{-1/2})$.

6. Suppose that the link $L$ can be represented by a diagram $L_1 \cup L_2$ with links $L_1$ and $L_2$ that are on different sides of some vertical line in the plane. Prove that the Conway polynomial $\nabla_L(z) = 0$.

[See the reverse side for hints!]
Hints:

3. What effect does taking the mirror image have on (a) a state diagram? or (b) a resolving tree?

4. Think about how changing the orientation of a knot might affect its resolving tree.

5. Some of these substitutions take place in the complex number system, where \(i^{-1} = -i\) for \(i = \sqrt{-1}\). You only have to show that the skein relation of one polynomial becomes the skein relation of the other polynomial under the indicated substitution.

6. Interpret the diagram \(L_1 \cup L_2\) as the smoothing \(L_s\) of a diagram \(L_+\) which equals its diagram \(L_-\). Do this by forming a connected sum of the two links \(L_1\) and \(L_2\) with one extra crossing in between. Now apply the skein relation for \(\nabla(z)\).