Chapter 3

#5 Follow the example given in the text, right below the problem.

#6 Proceed similarly to Problem 5. That is, translate towards the origin, scale, and translate back.

#8 Follow the inversion construction of your class notes.

#11 Start with \( P = (x, y, h) \) and \( Q = (-x, -y, -h) \). Project both \( P \) and \( Q \) stereographically, according to the formula

\[
S(x, y, h) = \frac{x + yi}{1 - h}.
\]

Then take \( z = S(P) \) and \( w = S(Q) \) and compute \( z\bar{w} \). Remember that \( x^2 + y^2 + h^2 = 1 \).

#12

\[
a = \frac{2x}{x^2 + y^2 + 1} \\
b = \frac{2y}{x^2 + y^2 + 1} \\
c = \frac{x^2 + y^2 - 1}{x^2 + y^2 + 1}
\]

#13 What the textbook calls the \( abc \)-coordinate system, we called the \( xyh \)-coordinate system in class. So, rotation of the Riemann sphere by 180° about the \( x \)-axis means inversion \( T(z) = 1/z \) of the complex plane. You want to describe a rotation of the Riemann sphere by 180° about the \( y \)-axis. You know the effect on the complex plane if the Riemann sphere is rotated about the \( h \)-axis. So, in order to rotate the sphere about the \( y \)-axis, first rotate the sphere about the \( h \)-axis by an appropriate angle, then about the \( x \)-axis, and then about the \( h \)-axis, again.

#15 (b) Write

\[
f(z) = \frac{2z - 1}{z + 1} = \frac{2 - \frac{1}{z}}{1 + \frac{1}{z}}
\]

and consider the limit as \(|z| \to \infty\), which means that \(1/z \to 0\).