#2 Check your notes. We did this problem in class.

#4 Switch to the upper half plane model. There, the configuration consists of a horizontal curve $C$ (which is a Euclidean straight segment) of hyperbolic length $d$ and two vertical hyperbolic straight lines $l_1$ and $l_2$. The area in question is enclosed above the horizontal. Let $y_0$ be the $y$-coordinate of the horizontal and $x_1$ and $x_2$ the $x$-coordinate of the vertical lines. Use the formula

$$A = \int_{x_1}^{x_2} \int_{y_0}^{\infty} \frac{1}{y^2} \, dy \, dx$$

to compute the area of this region. Then use the formula

$$d = l(C) = \int_a^b \frac{1}{y(t)} |z'(t)| \, dt$$

to compute the length of the horizontal curve $C : z = z(t)$.

Show that $A = d$.

#5 Say the hyperbolic circle has hyperbolic center $p$ and hyperbolic radius $R$. Find a transformation $T \in \mathbb{H}$ such that $T(p) = 0$. Explain why the hyperbolic circle turns into a Euclidean circle centered at 0. Let $r_0$ be the Euclidean radius of this circle centered at the origin. Explain why

$$R = \ln \left( \frac{1 + r_0}{1 - r_0} \right).$$

Solve this equation for $r_0$ (you’ll get $r_0 = \tanh(R/2)$) and compute the area of the circle with the formula

$$A = \int_0^{2\pi} \int_0^{r_0} \left( \frac{2}{1 - r^2} \right)^2 r \, dr \, d\theta.$$

Finally, check if your answer agrees with $A = 4\pi \sinh^2(R/2)$. (While you can translate everything into exponential functions, the problem becomes easier when you use hyperbolic identities throughout.)
#8 Suppose a rectangle, that is, a figure bounded by 4 hyperbolic straight segments at 4 right angles, existed in hyperbolic geometry. Draw any one of its diagonals and ponder on the obtained triangles.

For the second part of the question: draw an equiangular figure, centered at the origin, of some random area and enlarge (or shrink) to the desired area. (This procedure will change the angles. However, every angle will change by the same amount.)

Note: This figure is only unique if we also require all four sides to have the same length.

#12 Suppose you start with a hyperbolic straight line. What kind of a curve would be equidistant to it? Would it be a hyperbolic straight line?

#13 Sketch some drawings in both models of hyperbolic geometry and comment on your constructions from a railway engineering point of view.