Chapter 12

§12.1. 10. Does the Pythagorean Theorem hold?
11. Check to see if $|AB| + |BC| = |AC|$.
12. Locate the point $Q$ closest to $P(4, -2, 6)$ on the given object and compute their distance $d(P, Q) = |PQ|$.
18. Complete the squares. (Follow Example 6.)

§12.2. 36. Follow Example 7.
50. Start with $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ and compute both sides of Property 5. (Mind the parentheses!)
Then sketch the vector addition of $a + b$ (as in Figure 3) and superimpose that of $ca + cb$. Explain why you arrive at similar triangles and use the similar triangles to deduce Property 5.

§12.3. 38. Note that there are two solutions for $\gamma$.
58. Let $\alpha$ be the angle between $a$ and $c$ and let $\beta$ be the angle between $b$ and $c$. You wish to prove that $\alpha = \beta$. Since both angles are between 0 and 180$^\circ$, you only have to show that $\cos \alpha = \cos \beta$. So, compute

$$\cos \alpha = \frac{c \cdot a}{||c|| ||a||} = \cdots$$
$$\cos \beta = \frac{c \cdot b}{||c|| ||b||} = \cdots$$

60. Let the parallelogram be spanned by the vectors $a$ and $b$. Express the two diagonal vectors in terms of $a$ and $b$ and compute their dot product. (Make use of the properties of the dot product, as summarized in [2].)

§12.4. 28. The parallelogram is spanned by $a = \overrightarrow{PQ}$ and $b = \overrightarrow{PS}$. Use the fact that the length of the cross product is equal to the area of the parallelogram.
33. Use Formula [14].
37. Three vectors are coplanar exactly if the parallelepiped which they span has volume equal to zero.
45. First draw a sketch. Consider the parallelogram spanned by \( \mathbf{a} \) and \( \mathbf{b} \). There are two ways to compute its area: (1) It equals the length of the cross product; (2) You can multiply \((\text{BASE}) \times (\text{HEIGHT})\), where \( \mathbf{a} \) constitutes the base and the distance \( d = d(P, L) \) from \( P \) to the line \( L \) is your height.

50. Let \( \mathbf{a} = (a_1, a_2, a_3) \), \( \mathbf{b} = (b_1, b_2, b_3) \), and \( \mathbf{c} = (c_1, c_2, c_3) \). Work out both sides and compare.

§12.5.

5. Use the normal to the plane as the directional vector for the line. Note that you are asked to find both the vector equation (Equation 1) and the parametric equations (Equations 2).

7. The two points \( P(0, 1/2, 1) \) and \( Q(2, 1, -3) \) are given to be on the line. So, \( d = \overrightarrow{PQ} \) can serve as a directional vector. Write your answer in the forms of Equations 2 and Equations 3.

21. As we did in class, convert the given symmetric scalar equations to parametric equations. Make sure to use a different time parameter for each of the two lines before you set them equal. Follow Example 3.

27. To create a parallel plane, use the same normal.

33. Three points on the plane are given: \( P(2, 1, 2) \), \( Q(3, -8, 6) \) and \( R(-2, -3, 1) \). So, the plane is spanned by the vectors \( \mathbf{a} = \overrightarrow{PQ} \) and \( \mathbf{b} = \overrightarrow{PR} \). Therefore, \( \mathbf{n} = \mathbf{a} \times \mathbf{b} \) can serve as a normal.

48. Find the parametric scalar equations for the line and plug them into the equation for the plane. Then solve for the parameter \( t \) and specify the intersection point.

53. The angle between two nonparallel planes is the same as the angle between their normal vectors.

60. First you need to locate one point on the line \( l \), which consists of the intersection of the two planes \( p_1 \) and \( p_2 \). For example, set \( z = 0 \) and solve the system of two equations for the planes. Since \( l \) is on both planes, its direction vector must be perpendicular to both normals. Write down the two normals \( \mathbf{n}_1 \) and \( \mathbf{n}_2 \) and find a vector \( \mathbf{d} \) perpendicular to both. Then \( \mathbf{d} \) is a possible direction vector for the line.
69. Before you can use the formula of Problem 45 in Section 12.4 you have to generate two points $Q$ and $R$ on the line. Simply take two different values for $t$.

71. Use Formula [9].

75. The distance between two parallel planes $p_1$ and $p_2$ is the same as the distance from any point $P_1$ on plane $p_1$ to the plane $p_2$. Think about what happens to Formula [9] when you compute $d(P_1, p_2)$.

78. Follow the example from class. See also Example 10.

§12.6. 8. Sketch the graph in the $yz$-plane and then extend it in the $x$-direction.

20. For example, the traces where $y = k$ is fixed, are parabolas: $x = -z^2 + k^2$.

37. Complete the square and put everything in the form

$$\pm \frac{(x - x_0)^2}{a^2} \pm \frac{(y - y_0)^2}{b^2} \pm \frac{(z - z_0)^2}{c^2} = 1.$$ 

Then $(x_0, y_0, z_0)$ is the center of the figure and its type can be found in Table 1. Make sure to identify the symmetry axis by matching the variables correctly.

46. Draw a sketch. The figure is a cone whose axis is the $z$-axis. The traces $z = k$ are circles of the form $x^2 + y^2 = r^2$. Figure out the radius in terms of $z$ and you are done.

47. Start with a generic point $P(x, y, z)$. Let $Q$ be the point $Q(-1, 0, 0)$ and let $p$ be the plane $x - 1 = 0$. You are supposed to find all points $P$ for which $d(P, Q) = d(P, p)$.

Recall that

$$d(P, Q) = \sqrt{(x + 1)^2 + (y - 0)^2 + (z - 0)^2}$$

and that

$$d(P, p) = |1(x) + 0(y) + 0(z) - 1|/\sqrt{1^2 + 0^2 + 0^2}.$$

There is your equation. Simplify and identify it.

51. Use the fact that $P(a, b, c)$ is on the hyperbolic paraboloid $z = y^2 - x^2$ to verify that both sets of linear parametric equations satisfy the equation of the surface.