ABSTRACTS

Multidesigns of the λ-fold complete graph

Atif Abueida, University of Dayton

By a *graph-pair of order* t, we mean two non-isomorphic graphs G and H on t non-isolated vertices for which $G \cup H \cong K_t$ for some integer $t \geq 4$. we consider the existence of multidecompositions of λK_m for the graph-pairs of order 4 or 5. For those graph-pairs, we will also look for maximum multipackings and minimum multicoverings of λK_m.

An Upper Bound of Integrity of n-cage Graph

Mustafa Atici, Western Kentucky University

Integrity, a measure of network to reliability, is defined as $I(G) = \min_{S \subset V} \{|S| + m(G - S)\}$, where G is a graph with vertex set V and $m(G - S)$ denotes the order of the largest component of $G - S$. We prove an upper bound of the following form on the integrity of any r-cage graph with n vertices:

$I(G) \leq \frac{n+2}{4} + I(P_{\frac{3n-2}{4}})$, if $\frac{n+2}{4} = 4t$

$I(G) \leq \frac{n}{4} + I(C_{\frac{3n}{4}})$if $\frac{n}{4} = 4t'$

where t and t' are integers. To prove this upper bound we first solve the following problem: Suppose G is a connected cubic graph with $g \geq 2(d - 3)$ and $d \geq 5$, where d is *diameter* and g is *girth* of the graph G. We can remove $\lceil \frac{n}{4} \rceil$ vertices from G in such a way that the remaining graph is a tree or a $G(\frac{3n}{4}, \frac{3n}{4})$ graph.

On Rainbow Arithmetic Progressions

Maria Axenovich*, Iowa State University
Dmitri Fon-Der-Flaass

Consider natural numbers $\{1, \cdots, n\}$ colored in three colors. We prove that if each color appears on at least $(n + 4)/6$ numbers then there is a three-term arithmetic progression whose elements are colored in distinct colors. We also discuss a many-color version of this result providing a generalization of Van der Waerden's theorem.

A new construction for the n-cube partitioning

Sergei Bezrukov, University of Wisconsin-Superior

We consider the problem of partitioning the vertex set of the n-cube into k subsets $A_1, \ldots, A_k$ so that the cardinalities of these subsets differ at most on one. The objective it to minimize the total number of edges that connect the vertices of different subsets. Our new result is a construction which works for the values of k of the form $k = (2^a - 1)(2^{abc} - 1)/(2^{bc} - 1)$ where $a > 0$, $b > 1$, $c > 0$ are integers. This series includes all cases for which an optimal partitioning is known.

When a cartesian product is a triangle graph

Weiting Cao, University of Illinois at Urbana-Champaign

The triangle graph is a generalization of the concept of line graph. The triangle graph of a graph G, denoted by $T(G)$, is the graph whose vertex set is the set of triangles of G, with two vertices adjacent in $T(G)$ if they have a common edge in G. We prove the following result: For $n \geq 2$, the cartesian product of G and K_n is a triangle graph if and only if G is the line graph of some triangle-free graph.

Isometric decomposition of complete graphs

Harry Calkins Wolfram Research
Roger B. Eggleton*, Illinois State University

Abstract: Suppose the graph G is a proper divisor of the complete graph K_n, so K_n has a decomposition into$r \geq 2$ edge-disjoint copies of G. If X is a given representation of KK_n in $d-$dimensional Euclidean space, does X admit a decomposition into r edge-disjoint copies of G which are isometric to each other? We present a number of recent results on this isometric decomposition problem.

List-Coloring Certain Complete Multipartite Graphs

Linda Eroh, University of Wisconsin Oshkosh

We will discuss several minor results about list-coloring certain complete multipartite graphs. Let $K_r(n)$ represent the complete n-partite graph in which each partite set has r vertices. We show that $K_2(n)$ is n-list-colorable and $K_3(n)$ is not n-list-colorable. We will also see some partial results concerning which graphs with n partite sets, some of order 2 and some of order 3, are n-list-colorable.

Regular Clique Covers of Graphs

Dalibor Fronček, University of Minnesota Duluth and Technical University Ostrava

A family of cliques in a graph G is said to be p-regular if any two cliques in the family intersect in exactly p vertices. A graph G is said to have a p-regular k-clique cover if there is a p-regular family H of k-cliques of G such that each edge of G belongs to a clique in H. Such a p-regular k-clique cover is separable if the complete subgraphs of order p that arise as intersections of pairs of distinct cliques of H are mutually vertex-disjoint.

For any given integers p, k, ℓ; $p < k$, we present bounds on the smallest order of a graph that has a p-regular k-clique cover with exactly ℓ cliques, and we describe all graphs that have p-regular separable k-clique covers with ℓ cliques.

Graphs of Equivalent Lattice Paths

Rick Gillman, Valparaiso University

A new family of graphs is defined on set of paths on the mxn lattice. The paths themselves form the vertex set, and two are connected by an edge if they share more than k path steps in the lattice. Motivation for investigating these graphs will be provided, as well as various elementary properties.

On k-wise set-intersections

Weiting Cao, University of Illinois
Kyung-Won Hwang*, University of Illinois

We prove a version of the Ray-Chaudhuri-Wilson and Frankl-Wilson theorems for k-wise intersections and give a better upper bound on size of set-systems. Our result is as follows: Let L be a set of non-negative integers of size s. Let $k \geq 2$ be an integer and let $\mathcal{H}$ be a family of n-element set such that $|H| \notin L$ for any $H \in \mathcal{H}$ and $|H_1 \cap H_2 \cap ... \cap H_k| \in L$ for any collection of k distinct sets from $\mathcal{H}$. Then we have

$$|\mathcal{H}| \leq (k-1) \sum_{i=0}^{s} \binom{n-1}{i}$$

Furthermore, if $L = \{0, 1, 2, ..., s-1\}$, then we have

$$|\mathcal{H}| \leq (k-1) \binom{n}{s}$$

Fast necessary criteria for vertex-transitivity of graphs

Robert Jajcay*, Indiana State University
Sandi Malnic
Dragan Marusic

Determining the vertex-transitivity of a graph is generally a hard task (although the precise complexity of this problem is not known). Any "fast" arithmetic tests for vertex-transitivity are therefore of great interest.

In our talk, we generalize the results of J. Širáň and the first author (Australasian J. Combin. **10** (1994)), and develop new formulas for the number of closed walks of length p^r or pq, where p and q are primes, valid for all vertex-transitive graphs. Based on these formulas, several simple tests for vertex-transitivity are presented, as well as lower bounds on the orders of the smallest vertex- and arc-transitive groups of automorphisms for vertex-transitive graphs of given valence.

We demonstrate the use of our methods by applying them to the class of generalized Petersen graphs.

Some Polyomino Packing Games

Jerry Kabell, Central Michigan University

Consider the following game scenario. The playing board is an m X n rectangular grid. Two players alternatively place polyomino shaped pieces on the board, precisely covering a corresponding set of cells. The last player to successfully place a piece is the winner. We investigate the existence of winning strategies under a variety of conditions, including choices of m and n, and choices of the permissible polyominoes.

Covering n-space and Coloring metric spaces

Zoltán Füredi, University of Illinois at Urbana-Champaign
Jeong-Hyun Kang* , University of Illinois at Urbana-Champaign

Rogers [1957] proved that for every closed convex body C in $\mathbf{R}^n$, there is a covering of $\mathbf{R}^n$ by translates of C that has density at most $O(n \ln n)$. However, a covering with low global density can have high multiplicity, where the *multiplicity* is the maximum number of copies of C covering a single point. Erdős and Rogers [1962] showed that, for sufficiently large n, there is a covering of $\mathbf{R}^n$ by translates of C that has density at most $O(n \ln n)$ and multiplicity at most $O(n \ln n)$. In this talk, we give a combinatorial proof of this using the Local Lemma.

We also give an application of this theorem. Let G be the graph on $\mathbf{R}^n$ in which points are adjacent if their distance is 1 in the l_p norm. Kang and Füredi proved that G has chromatic number between $(1.067)^n$ and $\sqrt{p/(2\pi n)}(5(ep)^{1/p})^n$. We apply the theorem above to obtain an upper bound of $c(n \ln n)5^n$ on the chromatic number of G, independent of p. This simplifies the previous upper bound argument and improves the upper bound when p is not too large.

Piercing numbers and coloring disjointness graphs of translates of a convex set

Seog-Jin Kim, University of Illinois at Urbana-Champaign

Let F be a family of translates of a fixed convex set M in the plane. Let $\tau(F)$ be the minimum number of points piercing all members of F, and let $\nu(F)$ be the maximum size of a pairwise disjoint subfamily of F. We will show that $\nu \leq \tau(F) \leq 8\nu$. Furthermore, if M is centrally symmetric, then $\nu \leq \tau(F) \leq 6\nu$. Let G be the complement of the intersection graph of F; that is, $V(G) = F$, and two members of F are adjacent if and only if they are disjoint. We will show that $\chi(G) \leq 3\nu - 2$.

Vertex magic total labeling of Cartesian products of some regular VMT regular graphs and even cycles

Petr Kovář, University of Minnesota Duluth and VSB - Technical University of Ostrava

A vertex-magic total labeling of a graph $G(V, E)$ is defined as one-to-one mapping from the set of integers $\{1, 2, ..., |V| + |E|\}$ to $V \cup E$ with the property that the sum of the label of a vertex and the labels of all edges incident to this vertex is the same constant for all vertices of the graph. In the talk we present some techniques for constructing vertex magic total labelings of Cartesian products of even cycles and some vertex magic total regular graphs with additional properties.

Diameters of isomorphic spanning trees factorizing complete graphs

Dalibor Fronček, University of Minnesota Duluth and Technical University Ostrava
Tereza Kovářová*, University of Minnesota Duluth and VSB - Technical University of Ostrava

It has been proved that complete graphs K_{2n} for $n \equiv 1, 3\,(mod\,4)$ can be factorized into n copies of a spanning tree of any diameter d, where $3 \leq d \leq 2n - 1$. We will present a method for factorization of an infinite subclass of complete graphs K_{2n} where $n \equiv 0, 2\,(mod\,4)$. The method enables construction of isomorphic spanning trees of all diameters d, $3 \leq d \leq 2n - 1$.

Edge coloring of simple graphs with small maximum degrees

Xuechao Li
Rong Luo*, Middle Tennessee State University
Jianbin Niu
Xiaodong Zhang

In this paper, we first give new lower bounds for the size of Δ-critical graphs with $6 \leq \Delta \leq 8$ and then, by applying these lower bounds, we prove that every simple graph with maximum degree at least 8 embedded in the surface with Euler characteristic -1 is Class one.

You've got Erdos-Ko-Rado: Intersecting hypergraphs online

T. Bohman,
C. Cooper,
A. Frieze
Ryan Martin *, Iowa State University
M. Ruszinko

Let c be a positive constant. We show that if $r = \lfloor cn^{1/3} \rfloor$ and the members of $\binom{[n]}{r}$ are chosen sequentially at random to form an intersecting hypergraph then with limiting probability $(1+c^3)^{-1}$, as $n \to \infty$, the resulting family will be of maximum size $\binom{n-1}{r-1}$.

Minimal Separators versus Maximal Bicliques

Terry McKee, Wright State University

I present a simple one-way relationship between "minimal vertex separators" and "maximal complete bipartite subgraphs" involving both set complementation and graph complementation. This relationship has ramifications for chordal graphs and for two other similarly-parallel concepts. There is also a corresponding two-way relationship between multiple vertex separation and complete multipartite subgraphs.

Equitable Δ-coloring of graphs with low average degree

Alexander Kostochka, University of Illinois at Urbana-Champaign
Kittikorn Nakprasit*, University of Illinois at Urban-Champaign

An *equitable coloring* of a graph is a proper vertex coloring such that the sizes of any two color classes differ by at most 1. A graph may have an equitable k-coloring but not have an equitable $(k+1)$-coloring; for example, $K_{7,7}$ is equitably 2-, 4-, and 6-colorable, but not equitably 3-, 5-, or 7-colorable. Let $\chi_{eq}(G)$ denote the smallest number m such that for each $k \geq m$, G has an equitable k-coloring.

Hajnal and Szemerédi [1970] proved that every graph G has an equitable k-coloring for each $k \geq \Delta(G)+1$. This deep result is sharp, but the bound can be improved for some important classes of graphs. Chen, Lih, and Wu conjectured that $\chi_{eq}(G) \leq \Delta(G)$ for every connected graph G except complete graphs, odd cycles, and complete bipartite graphs with part-sizes that are equal and odd. They proved their conjecture for graphs with maximum degree 3. The conjecture is also proved for bipartite graphs, interval graphs, 2-degenerate graphs, and some other families of graphs.

The aim of the talk is to prove Chen–Lih–Wu Conjecture for graphs with low average degree. Given $\Delta \geq 36$, we show that $\chi_{eq}(G) \leq \Delta(G)$ for every $K_{\Delta+1}$-free graph G with maximum degree Δ and average degree at most $\Delta/6$

Functions of the degree sequence that give lower bounds on independence number

Patricia Nelson, University of Wisconsin-La Crosse

There are many functions of the degree sequence of a graph which are known to give lower bounds on the independence number of the graph. We compare three of these known bounds to each other and give a construction which shows that one of them, the residue, is asymptotically better than the other two under some circumstances. More precise estimates are possible when one takes into account other details of the graph, such as girth.

Chords of the longest circuits in locally planar graphs

Jianbing Niu, West Virginia Unievrsity

It was conjectured by Thomassen (1985) that every longest circuit of a 3-connected graph must have a chord. This conjecture is verified for locally planar graphs in this paper that there is a function $h : N \longrightarrow N$, such that, for every 4-connected graph G with minimum degree at least five embedded in a surface with Euler genus g and face-width at least $h(g)$, every longest circuit of G has a chord.

On Recovering a Multipartite Tournament from its Lattice of Convex subsets

Darren Parker, University of Dayton

The collection of convex subsets of a multipartite tournament T forms a lattice (T). Given a lattice structure for (T), we deduce properties of T. In particular, we find conditions under which we can detect clones in T (i.e. vertices with identical arc orientations). We also determine conditions on the lattice which will imply that T is bipartite, except for a few cases. We classify the ambiguous cases. Finally, we study a property of (T) we call the *anti-bipartite condition.* We prove a result on directed cycles in multipartite tournaments satisfying the anti-bipartite condition, and determine the minimum number of partitions in such multipartite tournaments.

The toughness of a toroidal graph with connectivity 3 and girth 6 is exactly 1

Weiting Cao, , University of Illinois at Urbana-Champaign
Michael J. Pelsmajer*, Illinois Institute of Technology

We show that any toroidal graph with connectivity 3 and girth 6 is bipartite. This implies that its toughness is at most 1. This answers a question in Goddard, Plummer, and Swart [1997] in which it was shown that such a graph has toughness at least 1.

Reconstruction under group action

Jamie Radcliffe, University of Nebraska-Lincoln

Combinatorial reconstruction problems arise when we are given the subobjects of a certain size of some combinatorial object, up to isomorphism, and are asked whether this is sufficient information to reconstruct the original object. For instance the Reconstruction Conjecture, made sixty years ago by Ulam and Kelly, asserts that all finite graphs on at least 3 vertices can be reconstructed from the collection of all their (non-trivial) induced subgraphs. Similarly the Edge Reconstruction Conjecture asserts that every graph with at least 4 edges can be reconstructed from the collection of all its (non-trivial) subgraphs. Reconstruction problems have been considered for a variety of other combinatorial objects, including directed graphs, hypergraphs, infinite graphs, codes, sets of real numbers, sequences, and combinatorial geometries. The necessary ingredients for a combinatorial reconstruction problem are a notion of isomorphism and a notion of subobject. Some progress has been made in recent years in the general case where we have a group action providing the notion of isomorphism, and we wish to reconstruct a subset S of X from the multiset of isomorphism classes of its k-element subsets, known as the k-deck. I will discuss some of this progress.

Max-Coloring interval graphs

Sriram Pemmaraju, University of Iowa
Rajiv Raman*, University of Iowa
Kasturi Varadarajan, University of Iowa

Given a graph $G = (V, E)$ and positive integral vertex weights $w : V \to \mathbf{N}$, the *max-coloring problem* seeks to find a proper vertex coloring of G whose color classes $C_1, C_2, \ldots, C_k$, minimize $\sum_{i=1}^{k} max_{v \in C_i} w(v)$. This problem, restricted to interval graphs, arises whenever there is a need to design dedicated memory managers. The problem is NP-hard for interval graphs. Though this problem seems similar to the well-known dynamic storage allocation problem, there are fundamental differences. We make a connection between max-coloring and on-line graph coloring and use this to devise a simple 2-approximation algorithm for max-coloring on interval graphs. We also show that a simple first-fit strategy, that is a natural choice for this problem, yields a 10-approximation algorithm. We show this result by proving that the first-fit algorithm for on-line coloring an interval graph G uses no more than $10 \cdot \chi(G)$ colors, significantly improving the bound of $26 \cdot \chi(G)$ by Kierstead and Qin (*Discrete Math.*, 144, 1995).

Orienting Graphs to Optimize Acyclic Reachability

Suk Jai Seo, Middle Tennessee State University

Let (G, R) denote the directed graph obtained from undirected graph G by an acyclic orientation R of its edges so that (G, R) contains no directed cycle. We consider orientations R of a graph G which maximize/minimize the number of ordered pairs of non-adjacent vertices with directed paths in (G, R).

Hamiltonian Connectedness in 3-connected Line Graphs

HongJian Lai, West Virginia University
Yehong Shao*, West Virginia University
Gexin Yu, University of Illinois
Mingquan Zhan, Millersville University

We investigate graphs G such that the line graph $L(G)$ is hamiltonian connected if and only if $L(G)$ is 3-connected, and prove that if each essential 3-edge-cut contains an edge lying in a short cycle of G, then $L(G)$ has the above mentioned property. Our result extends Kriesell's result in [J. of Combinatorial Theory, Ser. B. 82 (2001), 306-315] that every 4-connected line graph of a claw free graph is hamiltonian connected. Another application of our main result shows that if $L(G)$ does not have an hourglass (a graph isomorphic to $K_5 - E(C_4)$, where C_4 is an cycle of length 4 in K_5) as an induced subgraph, and if every 3-cut of $L(G)$ is not independent, then $L(G)$ is hamiltonian connected if and only if $\kappa(L(G)) \geq 3$, which extends a former result by Broersma, Kriesell and Ryjáček ([J. Graph Theory, 37 (2001), 125-136]) that every 4-connected hourglass free line graph is hamiltonian connected.

The n-Queens Problem With Diagonal Constraints

W. Doug Weakley, Indiana University - Purdue University at Fort Wayne

The *n-queens problem* requires placing n queens on an $n \times n$ chessboard so that no two attack each other. It has long been known that a solution exists for all positive integers n except 2 and 3; some recent work has aimed at determining good bounds for the number of solutions for each $n \geq 4$. Also, solutions of particular forms have been studied. We examine the n-queens problem with constraints on the diagonals that the queens can occupy, asking how close to (respectively, how far from) the board's two long diagonals the set of queen squares can be. More precisely, for a solution S of the n-queens problem, let $M(S)$ be the maximum of the absolute values of the diagonal numbers of S, and $m(S)$ the minimum of those absolute values. For $n \geq 4$, let $F(n)$ be the minimum value of $M(S)$, and let $f(n)$ be the maximum value of $m(S)$, as S ranges over all solutions of the n-queens problem. We give linear bounds for $F(n)$ and $f(n)$, and evidence that the bounds are quite good. Surprisingly, it appears likely that for most $n \geq 4$ there exist solutions S with $M(S) = F(n)$ and $m(S) = f(n)$.

How often does Chvátal's Condition hold?

Alexandr V. Kostochka, University of Illinois at Urbana-Champaign
Douglas B. West*, University of Illinois at Urbana-Champaign

Chvátal's Condition is a sufficient condition for a spanning cycle in an n-vertex graph. The condition is that when the vertex degrees are $d_1, \dots, d_n$ in nondecreasing order, $i < n/2$ implies that $d_i > i$ or $d_{n-i} \geq n-i$. We prove that this condition cannot hold in both a graph and its complement, and we raise the problem of determining its asymptotic probability in the random graph with edge probability 1/2.

Generating triples

Todd Will, University of Wisconsin-La Crosse

The non-negative integer triple (a, b, c) has size $a + b + c$ and generates the triples $(a + 1, b, c)$, $(a, b + 1, c)$, and $(a, b, c + 1)$. We introduce a technique to determine the number of size $d - 1$ triples required to generate all size d triples.

Circular flows of graphs

Hongjian Lai
Rui Xu*, West Virgina University
C-Q Zhang

In this paper, we get some sufficient conditions for graphs with circular flow index less than 4. As an immediate corollary, we give a simple proof of a result obtained recently by Galluccio and Goddyn *(Combinatorica, 2002)*, and obtain a larger family of such graphs.

An extension of a Dirac-type bound for k-ordered hamiltonian graphs

Alexander Kostochka, University of Illinois at Urbana-Champaign
Gexin Yu*, University of Illinois at Urbana-Champaign

For $3 \leq k \leq n$, an n-vertex graph G is k-ordered hamiltonian, if for any k ordered vertices of G, there is a hamiltonian cycle containing the k vertices in the given order. Kierstead, Sárközy and Selkow [J. Graph Theory, 32(1999), no 1, 17-25] considered a Dirac-type bound and proved that for $n \geq 11k - 3$, the condition $\delta(G) \geq \lceil n/2 \rceil + \lfloor k/2 \rfloor - 1$ ensures that G is k-ordered hamiltonian, and the minimum degree condition is sharp. In this paper, we extend this sharp bound to $n \geq 7k + 2$.

Homogeneously Embedding Stratified Graphs in Stratified Graphs

Gary Chartrand, Western Michigan University
Donald W. VanderJagt, Grand Valley State University
Ping Zhang*, Western Michigan University

A 2-stratified graph G is a graph whose vertex set has been partitioned into two subsets, called the color classes of G. Two 2-stratified graphs G and H are isomorphic if there exists a color-preserving isomorphism ϕ from G to H. A 2-stratified graph G is said to be homogeneously embedded in a 2-stratified graph H if for every vertex x of G and every vertex y of H, where x and y are colored the same, there exists an induced 2-stratified subgraph H' of H containing y and a color-preserving isomorphism ϕ from G to H' such that $\phi(x) = y$. Some results in this area are presented.