# MIGHTY XXXV ABSTRACTS

September 27 – 28, 2002

Illinois State University

### Integrity of Regular Graphs and Integrity Graphs

Mustafa Atici, Western Kentucky University

The *integrity* of a graph $G$, $I(G)$, is defined by $I(G) = min_{S \subseteq V(G)}\{|S| + m(G-S)\}$ where $m(G-S)$ is the maximum order of the components of $G-S$. In general the integrity of $r$-regular graph is not known [**?**]. We answer the following question for special regular graphs. For any given two integers $p$ and $r$ such that $\frac{pr}{2}$ is an integer, is there a $r$-regular graph, say $G^*$, on $p$ vertices having size $q = \frac{pr}{2}$ such that

$$I(G(p, \tfrac{pr}{2})) \leq I(G^*)$$

for all $p$ and $r$?

### On an Extremal Problem on Hypergraphs

Sergei Bezrukov, University of Wisconsin - Superior

Let $H = (V, E)$ be a hypergraph and let $A \subseteq V$. Denote by $E(A)$ the set of all hypergraph edges spanned by the vertex set $A$. The problem is for a given integer $m$ to find a subset $A \subseteq V$, $|A| = m$, such that $|E(A)| \geq |E(B)|$ for any $B \subseteq V$, $|B| = m$. This is an analog of an edge-isoperimetric problem on graphs. We consider this problem for cartesian products of hypergraphs and extent some graphs methods to solve this problem on hypergraphs. As an application we present solution for several hypergraph series.

### Graph theory package now in Mathematica

Harry Calkins, Wolfram Research Inc.

The latest release of Mathematica includes a graph theory package with a variety of capabilities. This talk will introduce the package and demonstrate some of its features.

### Spanning Trails Containing Given Edges

Zhi-Hong Chen, Butler University

For a graph $G$, a trail $T$ is a vertex-edge alternating sequence $v_0, e_1, \cdots, e_{k-1}, v_{k-1}, e_k, v_k$. A trail $T$ is called spanning trail if $V(T) = V(G)$. A $(u, v)$ trail of $G$ is a trial in $G$ whose origin is $u$ and whose terminus is $v$. A $(e', e'')$-trail is a trail whose first edge is $e'$ and whose last edge is $e''$. In this talk, we will present results on the following problem: for a graph $G$ and an edge subset $X \subseteq E(G)$ with $|X| \leq r$, what is the minimum edge-connectivity on $G$ such that $G$ has a spanning

$(u, v)$-trail $T$ (or $(e', e'')$-trail) such that $X \subseteq E(T)$ for any $u, v$ in $G$ (or $e', e''$ in $E(G) - X$). In particular, we shall show that for $r \geq 3$, if $G$ is $r + 1$ edge-connected, then for any $X \subseteq E(G)$ with $|X| \leq r$, $G$ has a spanning $(u, v)$-trail such that $X \subseteq E(T)$ for any $u, v$ in $G$.

### The Strong Matching Number of a Random Graph

Lane Clark, Southern Illinois University Carbondale

The strong matching number $sm(G)$ of a graph $G$ is the maximum number of edges in $G$ that induces a matching in the graph. For fixed $0 < p < 1$, A. El Maftouhi and L.M. Gordones [Australas. J. Combin. 10 (1994), 97-104; MR 95g:05091] showed that $sm(G(n, p))$ is one of only a finite number of values for a.e. random graph $G(n, p)$. We show that, in fact, $sm(G(n, p))$ is one of only two possible values for a.e. $G(n, p)$; determine the probability of attaining each value; and find the limiting distribution of the number of maximum strong matchings in $G(n, p)$.

### What are the possible sets of edge-deleted eccentricities of a graph?

Linda Eroh*, John Koker, Hosien Moghadam, Steve Winters, University of Wisconsin Oshkosh

For a vertex $v$ in a graph $G$, the edge-deleted eccentricity of $v$ is the minimum, over all edges $e \in E(G)$, of the eccentricity of $v$ in $G - e$. This concept has been explored in previous papers by Koker, Moghadam, Winters, Karen Klemm, Kevin McDougal, and Shubhangi Stalder. In this talk, we consider the following question: Given a set $S$ of positive integers, does there exist a graph $G$ so that the set of edge-deleted eccentricities of the vertices of $G$ is precisely $S$? Koker, Winters, and McDougal showed that such a graph exists for any set of consecutive integers in which the smallest is at least 2 and the largest is at most twice the smallest. For edge-deleted eccentricities, unlike the usual eccentricities, the integers do not have to be consecutive. Provided that each gap in the given set of integers is followed by a string of consecutive integers at least as long as the gap, it is the set of edge-deleted eccentricities for some graph. We will also show individual examples in which the gap is longer than the string of consecutive integers following it. For any integer $n > 2$, there is a graph with edge-deleted eccentricities precisely $\{n, n + 2\}$. However, we conjecture that there is no graph with edge-deleted eccentricities precisely $\{k, l\}$ where $l > k + 2$. This is shown in the case where $G$ has a cut-vertex whose removal results in three or more components.

### Cyclic decompositions of complete graphs into spanning trees

Dalibor Fronček, University of Minnesota Duluth and Technical University Ostrava

Graph factorizations, most often these of complete graphs, have been extensively studied by many authors. It is not surprising that factorizations into isomorphic factors received special attention over the years. There are many results on factorizations of complete graphs into isomorphic trees of smaller order. Surprisingly enough, almost nothing has been published on factorizations into isomorphic spanning trees. While methods of such decompositions into symmetric trees have been known, we have recently developed a more general method based on new types of vertex labelling, called *flexible q-labelling* and *blended q-labelling*. These labellings are generalizations of labellings introduced by Rosa and Eldergill. We present several classes of trees that allow factorization of complete graphs with an even number of vertices. In this talk, we will present the methods as well as some new classes of trees with a blended $q$-labelling.

**The Multiplicity of Eigenvalues in the Adjacency Matrix of a Tree**

L. Leslie Gardner* and Krystina K. Leganza, University of Indianapolis

This paper studies the multiplicity of eigenvalues of the adjacency matrix of a tree. The main result is that the multiplicity of any eigenvalue $\lambda$ of a tree whose adjacency matrix has $\lambda$ as an eigenvalue can be specified exactly in terms of matchings of proper subgraphs of the tree to single vertices. These proper subgraphs are called minimal graphs and their adjacency matrices also have $\lambda$ as an eigenvalue. They are called minimal because they do not have other minimal graphs as subgraphs to which the matching concept can be applied to find the multiplicity of $\lambda$.

**Probabilistic results concerning Seymour's distance two conjecture**

Zachary Cohn, University of Chicago
Anant P. Godbole*, East Tennessee State University
Elizabeth Wright, Tulane University

Seymour's distance two conjecture states that in any digraph, there exists a vertex (a"Seymour vertex") that has at least as many neighbors at distance two as it does at distance one. We explore the validity of probabilistic versions of this conjecture that focus on (i) the existence of Seymour vertices; (ii) the number of Seymour vertices; and (iii) almost everywhere questions in random tournaments and general random digraphs.

**Even submatrices of zero-one matrices**

Joe Johnson* and Anant Godbole, East Tennessee State University

Consider an $n \times n$ zero-one matrix $A$. A submatrix of $A$ is said to be *even* if the sum of its entries is even. In this talk, we will consider general submatrices, but the bulk of the exposition will be restricted to $2 \times 2$ submatrices. The *maximum* number of even $2 \times 2$ submatrices of $A$ is clearly $\binom{n}{2}^2$, and corresponds to the matrix $A$ having all ones (or zeros), so a more interesting question, motivated by Turań numbers and Hadamard matrices, is that of the *minimum* number $M(n)$ of such matrices. It has recently been shown that $M(n) \geq \binom{n}{2}^2 - Bn^3$ for some constant $B$. In this talk we use a variety of probabilistic methods to show that if the matrix $A$ is obtained at random, then

$$P\left(E \geq \binom{n}{2}^2 - Cn^{5/2}\right) \to 1$$

as $n \to \infty$, where $E$ denotes the number of even $2 \times 2$ submatrices of $A$.

**Some Graph-Theoretic Models for Sequencing Operations in a Stateful System**

Jerald A. Kabell, Central Michigan University

We consider a system containing clients, servers, and resources, in which a server must use one or more instances of a resource to provide the necessary service to a client. The use of a resource may change its state, and the state of a resource may affect its availability for use. We wish to sequence the service operations so as to minimize the overall resource requirements. Upper and lower limits on the resource requirements are readily obtained, but are in general not equal. Beginning with some instances in which the lower limit is attainable, we investigate some graph-theoretic models in an attempt to improve our bounds, seek conditions under which the lower limit is attainable, and devise algorithms for generating resource-minimal sequencings of operations.

### $L(2,1)$-labelling and packing of bipartite graphs

Jeong-Hyun Kang, University of Illinois at Urbana-Champaign

A nonnegative-integer coloring $f$ of the vertices of a graph $G$ is an $L(2,1)$-*labelling* if $|f(u) - f(v)| \geq 2$ for each edge $uv$ and $|f(u) - f(v)| \geq 1$ for each pair $u, v \in V(G)$ at distance 2 apart. The *L(2,1)-labelling span* of $G$, denoted by $\lambda(G)$, is the smallest number $t$ such that $G$ has an $L(2,1)$-labelling using no label larger than $t$. Griggs and Yeh (1992) conjectured that always $\lambda(G) \leq (\Delta(G))^2$. We have proved this for 3-regular Hamiltonian graphs. In this talk, we prove the conjecture for the incidence graph of every projective plane. For this graph $G$, the value of $\lambda(G)$ is $\Delta^2 - \Delta$. Our proof uses a result about packing of bipartite graphs that is analogous to the result of Sauer and Spencer for packing of graphs in general.

### On the chromatic number of intersection graphs of convex sets on the plane

Seog-Jin Kim, University of Illinois at Urbana-Champaign

Let $G$ be the intersection graph of a finite family of convex sets obtained by translations of a fixed convex set in the plane. We show that the chromatic number of $G$ is at most $3k-2$, where $k$ is the clique number of $G$. Also we show that the chromatic number of an intersection graph $H$ of convex sets obtained by translations and scalings of a fixed convex set in the plane is at most $6k-6$, where $k$ is the clique number of $H$. This is Joint work with A.V. Kostochka and K. Nakprasit.

### On a theorem of Erdős, Rubin, and Taylor

Alexandr Kostochka, University of Illinois at Urbana-Champaign

Erdős, Rubin, and Taylor found a nice correspondence between the minimum order of a complete bipartite graph that is not $r$-choosable and the minimum number of edges in an $r$-uniform hypergraph that is not 2-colorable (in the ordinary sense). In this note we use their ideas to derive similar correspondences for complete $k$-partite graphs and complete $k$-uniform $k$-partite hypergraphs.

### A Linear Algorithm for Finding the Reinforcement Number of a Tree

Patrick Jones and Grzegorz Kubicki*, University of Louisville

A set D of verices of a graph is a dominating set if each vertex not in D is adjacent to at lest one vertex in D. A mimimum dominating set is a dominating set with minimum cardinality. This cardinality is called the domination number of a graph. The reinforcement number of a graph G is the smallest number of edges that need to be added to G to decrease its domination number. The paper provides a linear algorithm for finding the reinforcement number of an arbitrary tree, answering an open question posed by Hedetniemi.

**Longest paths in circular arc graphs**

Jenő Lehel, The University of Memphis

Longest paths of a connected graph are obviously pairwise intersecting. Gallai asked whether it is true that all longest paths share a common vertex of the graph. Examples show that such a perfect "Helly-theorem" does not hold in general. However, we proved that the answer is affirmative for interval graphs, and also for the slightly larger family of circular arc graphs. (Joint work with P. Balister, E. Győri, and R. Schelp)

**Connectivity Properties of Visibility Graphs and Contracted Visibility Graphs**

Jay S. Bagga, John W. Emert, and J. Michael McGrew*, Ball State University

Given a set of $n$ nonintersecting line segments in general position in the plane, the segment endpoint visibility graph (SEVG) has the $2n$ endpoints as vertices, and $xy$ is an edge if the line segment $xy$ is one of the given segments, or it does not intersect any of the given segments. SEVGs and other types of visibility graphs have been extensively studied. These have applications in computer vision, robotics, and similar other fields. It is useful to also examine the properties of the related contracted visibility graph (CVG) of a given SEVG, which is the graph obtained by contracting each obstacle of the SEVG to a vertex and defining an edge of the CVG to represent the fact that either endpoint of one obstacle sees either endpoint of the other obstacle. In this paper we examine some connectivity properties of SEVGs as well as those of the related CVGs. In particular we explore relationships among the three connectivity numbers $\kappa, \lambda$, and $\delta$ for SEVGs and their CVGs.

**Graph Orientations and Signings That Are Saturated with Alternating Cycles**

Terry McKee, Wright State University

Imagine directing the edges of a graph (each toward one end or the other) or signing the edges (so that each is positive or negative). Those graphs that have orientations or signings in which all the even-length cycles are alternating have simple characterizations. In particular, having such a signing implies having such an orientation, and such orientations and signings are pleasantly entangled with orientations in which all the even-length cycles are directed cycles.

**The Secretary Problem**

Michal Morayne, Wroclaw University of Technology

***Invited Speaker***

The classical secretary problem is the following decision problem. There are n candidates for a job; their suitability is linearly ordered; they appear before the interviewer in a random permutation of their suitability ordering; and on the basis of each interview the interviewer is able to determine the relative suitability of all candidates interviewed so far. The interviewer wishes to choose the candidate currently being interviewed in such a way that the probability of choosing the best candidate is maximized. The solution of this problem is well known. The problem itself has been modified in various ways to make it correspond to different real life situations.

We shall discuss the analogous on-line decision problem where the linear order among the candidates is replaced by a partial order. An optimal best-choice algorithm will be given in the case where the partial order corresponds to a complete binary tree.

We shall also compare the probability of success (choosing the root of the ordering) if two independent searches were performed and the first ended with a tree which is a subposet of the tree that the second search ended with. We prove that if both trees are binary the second search yields a higher probability of success.

We shall discuss the combinatorial questions that are the key to solving this type of problem, in particular the inequalities involving the number of embeddings of an arbitrary tree into a complete binary tree of given height.

## The Chromatic Number of the Plane

Michal Morayne, Wroclaw University of Technology
***Invited Speaker***

The chromatic number of the plane (or, more generally, of Euclidean n-space) is the minimal number of colors necessary to assign a color to every point in such a way that no two points at distance one get the same color. Simple argument shows that this number for the plane is at least 4 and not greater than 7. Surprisingly, these simple estimates have not been improved for the last half-century. Nevertheless some deep and interesting results have been obtained. We shall briefly discuss them. The analogous problem has been considered for subsets of Euclidean n-space, in particular for rational n-space. The chromatic number problems for the rational plane and the Euclidean plane were treated more or less in parallel, using completely different methods. We shall show a connection between the existence of a k-coloring of Euclidean n-space and the existence of a regular coloring (a so-called "patch coloring") of rational n-space.

We shall also consider colorings that are determined by their restrictions to any open set (so-called "rigid colorings") and we shall show the connection between this notion and some classical results about rational n-space.

## An explicit construction for a generalized Ramsey problem

Dhruv Mubayi, University of Illinois at Chicago

We construct an edge coloring of $K_n$ such that every copy of $K_4$ receives at least four colors on its edges. The number of colors used in the construction is $n^{1/2+o(1)}$. This (almost) settles a problem of Erdős and Gyárfás, and can be thought of as an extension of the colorings yielding lower bounds for $r_k(C_4)$.

## Equitable colorings of k-degenerate graph

A.V. Kostochka and Kittikorn Nakprasit*, University of Illinois at Urbana-Champaign
S.V. Pemmaraju, The University of Iowa

In many applications of graph colorings the sizes of color classes should not differ too much. A formalization of this requirement is the notion of equitable coloring. An equitable coloring is a proper vertex coloring such that the sizes of color classes differ by at most 1. A $k$-degenerate graph is a graph whose every subgraph has minimum degree at most $k$. For example, forests are 1-degenerate, outerplanar graphs are 2-degenerate, and planar graphs are 5-degenerate.

The main result of the talk is finding that if the maximum degree $\Delta$ of a $k$-degenerate graph is not too large ($\Delta \leq \frac{n}{15}$), then $G$ is equitably $m$-colorable for every $m \geq 16k$. A corollary of this result is a polynomial-time approximation algorithm for equitable coloring allowing to color every $k$-degenerate graph admitting an equitable $s$-coloring with at most $82ds$ colors.

### On the Breadth of Lattices of Convex Sets in Multipartite Tournaments

Darren Parker, University of Dayton

We consider the breadth of the lattice of convex subsets of general multipartite tournaments. We show that the breadth of the lattice of convex subsets of a clone-free tournament (i.e. a tournament where not two vertices have identical arc orientations) is at most one larger than the tournament's second largest partition. This leads to a tight upper bound on the breadth of a tournament based on the number of vertices. We then show that all tournaments of maximum breadth are either bipartite or tripartite with exactly one vertex in the third partition, and we classify all such tournaments. Finally, we determine all subsets of vertices (atoms) of tournaments of maximum breadth that represent the breadth of the lattice of convex subsets.

### Equitable Choosability for Graphs of Bounded Tree-width

Michael Pelsmajer, Illinois Institute of Technology

A graph is *equitably $k$-choosable* if for every assignment of lists of size $k$ to its vertices, there is a proper coloring chosen from the lists such that each color class has size at most $\lceil n/k \rceil$. This combines the notions of $k$-choosability and equitable $k$-colorability.

We seek the least $k$ such that $G$ is equitably $j$-choosable for all $j \geq k$. The answer has been determined in terms of maximum degree for several classes of graphs. We have an upper bound of $\Delta + 3w$ for graphs of maximum degree $\Delta$ and tree-width at most $w$. We also have better upper bounds for graphs with relatively small tree-width.

### Some Matching Problems in the Boolean Lattice

Ian Roberts, Northern Territory University Darwin

An unresolved conjecture in the Boolean Lattice is the Union-Closed Sets Conjecture: In any union-closed collection on a finite set there is an element in at least half of the sets. Work on this conjecture by the speaker has lead to the informal question: How to choose m k-sets so as to minimise the union-closure over all possible choices of m k-sets? The question is informal because the idea of "minimise" must be made much more precise.

Attempts to solve this question have lead to some new problems being posed related to matchings between collections of sets of different cardinalities in the Boolean Lattice. A statement of the problems and motivation for the problems will be given, including a correct definition of "minimise" as used above.

### An eigenvalue proof of the uniqueness of the Ramsey Graph for $K_4$ versus $K_4$

Allen J. Schwenk, Western Michigan University

It has been long known that the Ramsey number $R(K_4, K_4)$ is 18, and the lower bound is forced by a unique graph of order 17, the Paley Graph. We provide an eigenvalue analysis that demonstrates this uniqueness in a straightforward manner.

### Packing of Jacks

Jozef Skokan, University of Illinois at Urbana–Champaign

For a point $\boldsymbol{c} = (c_1, c_2, \ldots, c_k) \in \{1, 2, \ldots, n\}^k$ we define a *jack* $J(\boldsymbol{c})$ with *center* $\boldsymbol{c}$ as the set of points that differ from $\boldsymbol{c}$ in at most one coordinate. For $i$, $1 \leq i \leq k$, and fixed $c_1, c_2, \ldots, c_{i-1}, c_{i+1}, \ldots, c_k \in \{1, 2, \ldots, n\}$, we also define a *line* as a set of $n$ points of the form

$$\{(c_1, c_2, \ldots, c_{i-1}, x, c_{i+1}, \ldots, c_k), 1 \leq x \leq n\}.$$

The following problem was formulated by Laszló Székely: Let $LS(n, k)$ be the maximum cardinality of a system $\mathcal{J}$ of jacks for which

*(1)* no two distinct jacks share a common line, and

*(2)* $\bigcap_{i=1}^{k} J_i = \emptyset$ for all distinct jacks $J_1, \ldots, J_k \in \mathcal{J}$.

Clearly $LS(n, k) \leq n^{k-1}$ holds, but Székely conjectured that $LS(n, k)/n^{k-1}$ tends to 0 as $n \to \infty$. In this talk we describe tools that one needs to prove this conjecture.

This is a joint work with Vojtech Rödl (Emory University).

### Flow-difference functions of graphs

Daniel Slilaty, Wright State University

Let $G$ be a graph and $f$ an edge direction of $G$. If $\vec{C}$ denotes a polygon of $G$ with a given orientation, then let $f(\vec{C})$ be the number of edges that $f$ directs along the given orientation of $\vec{C}$ minus the number of edges $f$ directs against the given orientation. The quantity $f(\vec{C})$ is often called the *flow difference* of $f$ along $\vec{C}$. We will explore flow-difference functions and some of their relationships to graph colorings and imbeddings of graphs in surfaces.

### On Partial Vertex List Colourings

P.E. Haxell, University of Waterloo
Zsuzsanna Szaniszlo*, Valparaiso University

Let $g$ be a graph that is $s-$ choosable. Suppose a list $S(v)$ of size $t < s$ of positive integers is assigned to each vertex $v$. We improve the lower bound on the number of vertices that can be properly coloured from the given lists. We prove that at least $t/s$ of the vertices can be properly coloured in case $s$ is a multiple of $t$. This settles part of a conjecture of Albertson, Grossma and Haas.

**On the integrity and arc-integrity for orientations of the hypercube**

Chip Vandell, Indiana University Purdue University at Fort Wayne

The integrity of a digraph D, (I(D)), has been defined as the minimum over all subsets T of the vertex set of the quantity m(D-T) + —T— where m(D) is the order of the largest strong component of D. The arc-integrity is defined in a similar manner. In this talk the value of these parameters will be discussed for orientations of the hypercube.

**Jedit - A Java Graph Editor**

Jay Bagga and Elizabeth Vandenberg*, Ball State University

JGraph is a Java based system for drawing graphs and for running graph algorithms. A number of well-known graph algorithms are provided, including those for planarity testing and drawing planar graphs on a grid. The algorithms can be run with an animation feature where the user can see intermediate steps as the algorithm executes. The system is extensible in that new algorithms can be easily added.

The system is primarily designed to be a learning and teaching tool. The graphical user interface is menu-driven and allows for intuitive manipulation of graphs.

**The maximum dimension of a shallow graph**

Matthew Walsh, Indiana University-Purdue University at Fort Wayne

The *depth* of a graph $G$ is the maximum dimension of an even subgraph of $G$. Graphs with depth at most 1 are called *shallow*. In this talk I determine the spectrum of possible dimensions for the flowspace of a shallow graph.

**The smallest $k$-regular, $h$-edge-connected graphs without 1-factors**

John Ganci, Texas Instruments
Douglas B. West*, University of Illinois

It is well known that every $k$-regular, $k-1$-edge-connected graph of even order has a perfect matching. For each pair $(k, h)$ with $0 \leq h \leq k-2$, we determine the minimum order of a $k$-regular, $h$-edge-connected graph of even order that has no perfect matching.

For $h = k-2$, the answer is $(\alpha+1)k-2$, where $\alpha$ is the least odd integer greater than $k$. For $h = 1$, the answer is $3\alpha+1$ (except at $k = 4$). In general, the answer is $(\alpha+1)(t+2)-2$, where $t$ is $\lceil 2h/(k-h) \rceil$ when $k-h$ is even and is $\lceil (2h+2)/(k-h-1) \rceil$ when $k-h$ is odd.

**Group connectivity of squares of graphs**

Rui Xu, West Virginia University

In 1992, Jeager et al. introduced group connectivity as a new concept as a generalization of nowhere-zero flow of graphs. In this talk, we give a characterization of the graph whose square is $Z_3$-connected. It generalizes a result about the nowhere-zero 3-flow of squares of graphs.

### Supereulerian matroids

Mingquan Zhan, West Virginia University

Let $M$ be a matroid. $M$ is supereulerian if there exists $X \subseteq E(M)$ such that $M|X$ is a cycle and $r(X) = r(M)$. In this paper we show that if every cocircuit of a regular $M$ has size at least $\max\{6, \frac{r(M)-3}{4}\}$, then $M$ is supereulerian. This is a joint work with Dr. Hong-Jian Lai.

### The Regularity Lemma and Graph Embedding

Yi Zhao, University of Illinois at Chicago

A standard extremal graph theory problem is that of investigating sufficient conditions for embedding a graph H in another graph G. The Regularity Lemma of Szemeredi provides a powerful tool for attacking many embedding problems. In this talk, we survey some recent results on graph embedding based on the Regularity Lemma.

### Random Growth of Caterpillar Graphs

Gabriel Zimmer* and Anant Godbole, East Tennessee State University

Consider a caterpillar graph grown in the following manner: Start with a single vertex. Then let a vertex be added to the graph at each time interval, along with a corresponding "segment". We will define a probability distribution at each time interval so that further segments are placed so that either a segment (i) is a leg of an existing spinal vertex; or (ii) is a new spinal vertex, with equal probability. For example, if such a caterpillar has spinal vertices $\{A, B, C\}$ at time $t$, then the segment grown at time $t + 1$ is a leg of $A$, a leg of $B$, a leg of $C$, or a new spinal vertex $D$ with equal probability 1/4. Let the (random) length of the caterpillar on $n$ vertices be $L_n$. We prove that $P(L_n = k) = S(n, k)/B_n$, where $S(n, k)$ is the $k$th Stirling number of the second kind, and $B_n$ is the $n$th Bell number. We provide exact formulas for the expected length and variance of the length of the caterpillar; study the asymptotic values of these quantities; and explore the limiting distribution of $L_n$.