MIGHTY XXXIX
Ball State University

ABSTRACTS

Session 1

A Solitaire Game Played on 2-Colored Graphs
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We consider a solitaire graph game. Initialize G by independently assigning a color red or green to each vertex. Select any currently green vertex v in the graph remaining, change the color of each neighbor of v, and delete v. Repeat. Removing all vertices constitutes winning the game.

We develop necessary conditions for winnability, and broad cases in which those conditions are sufficient (including trees and hypercubes for easy examples). We give efficient algorithms for deciding whether a maximal outerplanar game is winnable, and in general for reducing the winnability question to that for blocks.

Lengths in Cycle Spaces and Such
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I discuss both *my* proof and *the* proof that the sum of all the members of the cycle space of an n-vertex, m-edge graph always equals $m2^{m-n}$, and so that their average length is always $m/2$. I conclude with open questions concerning lengths in the intersection of the cycle and the cocycle spaces.

My Favorite Voltage Graph
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A simple voltage graph is displayed, leading to an overview of topological graph theory by answering every question that can reasonably be asked about the regular complete tripartite graphs $K(n, n, n)$.

Multiple Towers of Hanoi with a Path Transition Graph
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The multiple towers of Hanoi puzzle with a path transition graph will be discussed for the cases of three and four posts. The solution to the three post version with transition graph P_3 is well known, but it provides much insight into the four post version

with transition graph P_4. The four post version was originally suggested by Paul K. Stockmeyer, who called it the four-in-a-row puzzle. There are four subproblems here depending on the choice of the source and destination posts. Stockmeyer determined the minimum number of moves to solve this puzzle for the case that the source and destination posts are the two end posts and the number of disks is at most six. We have determined the minimum number of moves to solve the puzzle for any choice of the source and destination posts when the number of disks is at most twelve.

We also have the following results on the four post puzzle:

- Inequalities involving the minimum number of moves to solve each of the four subproblems mentioned above for any number of disks.
- In all four subproblems the minimum number of moves to solve the puzzle is $O(a^n)$ for any $a > 1$, where n is the number of disks.
- The minimum number of moves to solve the puzzle for the case that the source and destination posts are the two end posts is the diameter of the configuration state graph for any n.
- An algorithm that provides good upper bounds on the minimum number of moves to solve each of the four subproblems mentioned above for any n.

Session 2A

Multidecomposition of the Complete Graph with a Hamilton Cycle Leave
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A *graph-pair of order t* is two non-isomorphic graphs G and H on t non-isolated vertices for which $G \cup H \cong K_t$ for some integer $t \geq 4$. Given a graph-pair (G, H), we say (G, H) divides K_m if the edges of K_m can be partitioned into copies of G and H with at least one copy of G and at least one copy of H. We will refer to this partition as a (G, H)-*multidecomposition*.

In this paper, we consider the existence of multidecompositions of $K_m - H$ for the graph-pairs of order 5 where H is a Hamilton cycle. For those graph-pairs, we will also look for maximum multipackings and minimum multicoverings of K_m.

The s-Hamiltonian Index
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For simple connected graphs that are neither paths or cycles and an integer $m \geq 0$, we define $h_s(G) = min\{m : L^m(G) \text{ is } s\text{-Hamiltonian}\}$ and $l(G) = max\{m : G$ has an

arc of length m that is not both of length 2 and in a $K_3\}$, where an arc in G is a path in G whose vertices have degree two in G. We prove that $h_s(G) \leq l + s + 1$.

Hamiltonian Line Graphs

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A vertex cut X of a graph G is essential if either $G - X$ is a K_1, or at least two components of $G - X$ have edges. Thomassen conjectured that every 4-connected line graph is hamiltonian. We proved that every 4-connected, essentially 14-connected line graph is hamiltonian.

Stratified Domination in Graphs

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A graph G is 2-stratified if its vertex set is partitioned into two classes, the red vertices and the blue vertices. Let F be a 2-stratified graph rooted at some blue vertex v. The F-domination number of a graph G is the minimum number of red vertices of G in a red-blue coloring of the vertices of G such that every blue vertex v of G belongs to a copy of F rooted at v. Some results concerning F-domination are presented.

A Survey of Gallai-type Theorems for Generalized Domination and Degree Parameters

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Let G be a graph with n vertices. A Gallai-type theorem has the form $x(G)+y(G) = n$ where $x(G)$ and $y(G)$ are parameters defined on the graph G. It will be shown that $\chi(G) + \Psi(G) \leq n$ where $\chi(G)$ is a generalized domination parameter and $\Psi(G)$ is a degree condition parameter or a generalized domination parameter. Some of the possible parameters presented in this talk are independence, domination, independent domination and inverse domination. The conditions presented are minimum degree, maximum degree and vertex connectivity. In each case necessaryand sufficient conditions for equality to hold are given and specific classes of graphs where equality holds are described.

Perfect Doublers

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For a graph G, let $\gamma(G)$ denote the minimum size of a vertex dominating set of G. It is easily seen that $\gamma(K_2 \times G) \leq 2 \cdot \gamma(G)$. A vertex dominating set D of a graph G is *efficient* if each vertex of G is covered exactly once by D. For a regular graph G with an efficient dominating set, say that G is a *perfect doubler* if $\gamma(K_2 \times G) = 2 \cdot \gamma(G)$.

It is not difficult to check that the cycle C_{3i} is a perfect doubler only for $i = 1, 2$. It is known that those binary hypercubes and ternary hypercubes that have efficient dominating sets are perfect doublers. We give further examples of perfect doublers, a sufficient condition for the property, and explore related ideas.

Fractionalizing Inverse Domination
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In 1991 Kulli and Sigarkanti defined the problem of inverse domination: given a graph G, determine the smallest size of a set S such that S dominates G and is disjoint from some minimum dominating set D in G. We examine two variations on this theme from the perspective of real-valued domination.

Session 2B

The Nonorientable Genus of Joins of Complete Graphs with Large Edgeless Graphs
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We show that for $n = 4$ and for $n \geq 6$, K_n has a nonorientable embedding in which all the faces are hamilton cycles. Moreover, when n is odd there is such an embedding that is 2-face-colorable. Using these results we consider the join of an edgeless graph with a complete graph, $\overline{K_m} + K_n$, and show that for $n \geq 3$ and $m \geq n - 1$ its nonorientable genus is $\lceil (m-2)(n-2)/2 \rceil$ except when $(m, n) = (4, 5)$.

Relative Difference Sets Fixed by Inversion and Distance Regular Cayley Graphs
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I will present a connection between relative difference sets fixed by inversion and distance regular graphs and survey some known results of such Cayley graphs.

Tree-thickness and Caterpillar-thickness of Connected Graphs
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In 1978, Chung proved that every connected graph G with n vertices decomposes into at most $\lceil n/2 \rceil$ trees. We prove that a connected graph with n vertices and girth g decomposes into at most $\lfloor (n/g) + 1 \rfloor$ trees, if $g > 5$, and this is sharp. We prove weaker results when $g = 4$. A caterpillar is a tree having a single path incident to all edges. We prove that a connected outplanar graph G with girth 4 decomposes into at most $\lceil 3n/8 \rceil$ caterpillars, and this is sharp.

The Classical Convexity Numbers in Two-Path Convexity
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The most studied parameters in convexity spaces are the Helly, Radon, and Caratheodory numbers. Moreover, the hull number has gotten much attention in graph theory and can be defined on general convexity spaces as well. In this talk, we relate all four of these numbers to a lattice theoretic parameter called breadth (β). In the case that all singleton subsets are convex, we show that β is an upper bound for the Helly, Caratheodory, and hull numbers, and that $\beta + 1$ is an upper bound for the Radon number. We use this to find tight upper bounds for the Helly, Radon, Caratheodory, and hull numbers of clone-free multipartite tournaments in the context of two-path convexity. We have additional results concerning the Caratheodory number and clone-free bipartite tournaments.

Decomposing Facets of the Stable Set Polytope
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A simple graph is called α-critical if the deletion of any edge increases the maximum size of a stable set, called the stability number. Wade conjectured that in a connected, non-edge α-critical graph the deletion of two vertices will never decrease the stability number. Suranyi proved a weaker statement by showing that after deleting one vertex of degree at least 2, the remaining graph can be decomposed into α-critical subgraphs, none of which is an isolated vertex. We give a counterexample to Wade's conjecture, and generalize the decomposition result in a weaker form for the facets of the stable set polytope.

Decompositions of Signed Graphs
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Given two graphs G and H, a k-sum of G and H is obtained by identifying the graphs along a common k-clique and then deleting the edges of that clique. A classic theorem of Wagner's states that any graph with no K_5 minor can be obtained by 1, 2, and 3-sums of planar graphs, copies of $K_{3,3}$, and copies of V_8 (V_8 consists of an octagon along with four chords connecting antipodal vertices).

A signed graph is a pair (G, f) in which G is a graph and f is a labeling of the edges of G with elements of the multiplicative group {+1,-1}. In my talk we will define what minors of signed graphs are; what 1, 2, and 3-sums of two signed graphs are; and then give a Wagner-like decomposition theorem for an important class of signed graphs.

Equilateral Triangles in Finite Metric Spaces
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In the context of finite metric spaces with integer distances, we investigate the new Ramsey-type question of how many points can a space contain and yet be free of equilateral triangles (this is the analog of the classical problem on graph colorings and

monochromatic subgraphs, with the addition of the triangle inequality restriction). In particular, for finite metric spaces with distances in the set $\{1, \ldots, n\}$, the number D_n is defined as the least number of points the space must contain in order to be sure that there will be an equilateral triangle in it. Several issues related to these numbers are studied, mostly focusing on low values of n. Apart from the trivial $D_1 = 3$, $D_2 = 6$, we prove that $D_3 = 12$, $D_4 = 33$ and $81 \leq D_5 \leq 95$.

Session 3

Beineke and Matching Theory
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A *matching* in a graph G is a set of edges no two of which touch. A matching is *perfect* if it spans all the vertices of G. Just before the flood (in 1967 in fact), Lowell Beineke and the author published a paper in which we proved that an n-connected graph with a perfect matching must have at least n such. This result generalized a result of Kotzig, who had proved it for $n = 2$, although we didn't know it at the time.

Our result was itself improved and generalized in the years that followed by a number of authors including Zaks, Bollobás and Lovász among others. A related theory of canonical decompositions of graphs in terms of their maximum (or perfect) matchings continued to develop in the 1970's, further extending earlier work of Kotzig, Gallai and Edmonds. Today several branches of this theory remain quite active areas of research. I will briefly outline two of these:

1. graphs indecomposable with respect to this decomposition theory and
2. matching extension

Things I Learned at My Advisor's Knee: The Plot Thickens
Allen Schwenk, Western Michigan University, allen.schwenk@wmich.edu
We give a brief review of just a few of Lowell Beineke's many contributions to graph theory, with particular attention to thickness. Warning: the discussion might become coarse.

The Existence of a Rainbow 1-Factor in 1-Factorizations of Complete Uniform Hypergraphs
Mike Plantholt, Illinois State University, mikep@ilstu.edu
For positive integers $r \geq 2$ and n, the complete r-uniform hypergraph on set V of n vertices is the hypergraph $K_n^{(r)}$, with the edges consisting of all r-subsets of V. In 1977 Rosa asked if, for any given 1-factorization F of $K_{rn}^{(r)}$, $n \geq 3$, there exists a 1-factor whose edges belong to n different 1-factors of F. Woolbright and Fu provided an affirmative answer for simple graphs ($r = 2$), and made significant progress on the general problem. We give a proof of Rosa's general conjecture.

Beineke and The Nine Commandments
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We give a brief review of some of Lowell Beineke's many contributions to graph theory.
Session 4

New Families of Isoperimetric Graphs
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For a graph $G = (V, E)$ and a subset $A \subseteq V$ let $B(A)$ be the *ball* around A, i.e., $B(A) = \{v \in V \mid dist(v, A) \leq 1\}$. We consider a problem of finding for a given m, $1 \leq m \leq |V|$, a set $A \subseteq V$ such that $|A| = m$ and $|B(A)| \leq |B(A')|$ for any $A' \subseteq V$, $|A'| = m$. The set A is called *isoperimetric*.

The graph G is called *isoperimetric* if its vertex set admits a total order, such that any initial segment of this order is an isoperimetric set. Just a few infinite families of isoperimetric graphs are described in the literature. We present several new families of graphs, whose all cartesian powers admit isoperimetric orders.

On H-linked Graphs
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Let H be a fixed multigraph with vertices $w_1, \ldots, w_m$. A graph G is H-linked if for every choice of vertices $v_1, \ldots, v_m$ in G, there exists a subdivision of H in G such that v_i is the branch vertex representing w_i (for all i). The notion of H-linked graphs is a natural generalization of the notions of k-linked, k-ordered and k-connected graphs. In this talk, we explore various degree conditions on a graph G providing that G is H-linked. For a simple graph H with minimum degree at least 2, we give best possible Ore-type degree conditions, i.e., restrictions on the sum of degrees of pairs of non-adjacent vertices). For an arbitrary multigraph H, we give the best possible minimum degree conditions.

Connectivity of the (n, k)-Star Graph
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The star graph is a popular model considered as a topology for interconnection networks such as linking processors in a multiprocessor computer system. Originally proposed in the late 80s as a competitor to the n-cube, it has gained recognition and been studied in detail and generalized in many ways. In this paper we consider the directed (n, k)-star graph, a directed generalization of the star graph, and show that it is maximally connected.

The Size of Edge Chromatic Critical Graphs with Maximum Degree 6
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In 1968, Vizing conjectured that for any edge chromatic critical graph $G = (V, E)$ with maximum degree Δ, $|E| \geq \frac{1}{2}\{(\Delta - 1)|V| + 3\}$. This conjecture has been verified for $\Delta \leq 5$. By applying the discharging method, we prove the conjecture for $\Delta = 6$.

Edge Disjoint Partitions of Complete Bipartite Graphs
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We consider various edge disjoint partitions of complete bipartite graphs. One case is where we decompose the edge set into edge disjoint paths of increasing lengths. A graph G is *path-perfect* if there is a positive integer n such that the edge set $E(G)$ of the graph G can be partitioned into paths of length $1, 2, 3, \ldots, n$. The main result of the paper is the proof of Fink and Straight's conjecture: A complete bipartite graph $K_{s,t}$ on $t + s$ vertices ($t \leq s$) is path-perfect if and only if there is a positive integer n such that the following two conditions are satisfied:

(i) $st = 1 + 2 + \cdots + n = \binom{n+1}{2}$, and

(ii) $n \leq 2t$.

Our proof shows that the algorithm to find an edge disjoint partition of a complete bipartite graph into paths of lengths $1, 2, \ldots, n$ needs linear time to complete the process.

The Induced Matching Extendable Graphs
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A matching M is *induced* if $E(V(M)) = M$. We say that a graph G is *induced matching extendable*, shortly IM-*extendable*, if every induced matching M of G is included in a perfect matching of G. We get the following results:

1. The induced matching extendability of claw-free graphs of diameter 2.

2. The induced matching extendability of the composition of two graphs.

3. Characterization of the induced matching extendable graphs with $2n$ vertices and $3n - 1$ edges.

4. Characterization of the induced matching extendable graphs with $2n$ vertices and $3n$ edges.

5. Characterization of the induced matching extendability of the square of trees.

On Sparse Graphs with Large Numbers of Spanning Trees

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Let $t(G)$ denote the number of labeled spanning trees of a connected graph G. It is known how to compute $t(G)$. However, little is known about the extremal version of the problem, that is, given the number of vertices n and the number of edges m, find a connected (n, m) graph G such that $t(G) \geq t(H)$, where H is any other (n, m) connected graph. Such a graph G is called a t-optimal graph. In this presentation, we will give a summary of the known results on t-optimal graphs. We will then present some new results for the case of $(n, n + 4)$ t-optimal graphs. A demography of t-optimal graphs of order ≤ 12 which we have obtained by using a software called *nauty* will also be presented.

The spectra of super line multigraphs

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For an arbitrary simple graph G and a positive integer k, the super line multigraph of index k of G, denoted $L_k(G)$, has for vertices all the k-subsets of edges. Two vertices S and T are joined by as many edges as pairs of edges $s \in S$ and $t \in T$ share a common vertex. We give a formula to find the adjacency matrix of $L_k(G)$. If G is a regular graph, we calculate all the eigenvalues of $L_k(G)$ and their multiplicities.