MIGHTY XXXI

MIdwest GrapH TheorY Conference

Saturday, April 10, 1999
Oakland University
Rochester, Michigan

Program and Abstracts

Organizing Committee: Eddie Cheng, Curt Chipman,
Jerry Grossman, Marc Lipman (chair), Alan Park

Funding from the Oakland University Research Committee

Program

Friday, April 9, 1999

Reception at Jerry Grossman's home, 3125 Tamarron, Rochester Hills, 8–10 p.m.

Saturday, April 10, 1999

Talks are in Room 372, SEB. Speakers are shown in **bold font**.

Refreshments are in Room 376, SEB.

Talks are 15 minutes long, with 5 minutes for questions and transition.

8:45 a.m. coffee and registration

9:20 a.m. *Welcome*
Marc Lipman, Chair
Oakland University Department of Mathematics and Statistics

9:25 a.m. *Welcome*
Mary Papazian, Associate Dean
Oakland University College of Arts and Sciences

SESSION 1: Eddie Cheng, chair

9:30 a.m. *Geodetic Number of Random Graphs of Diameter 2*
Gab-Byung Chae, **Edgar M. Palmer**, and Wai-Cheong Siu
Michigan State University

9:50 a.m. *How Large Can the Domination Numbers of a Graph Be?*
Ping Zhang
Western Michigan University

10:10 a.m. *Hypergraph Decomposition and Large Scale Optimization*
H. Alan Park
Oakland University

10:30 a.m. coffee break

SESSION 2: Jerry Grossman, chair

11:00 a.m. *Characterizations for the Middle of a Graph*
Paul Brown and **Garry Johns**
Michigan State University and Saginaw Valley State University, respectively

11:20 a.m. *Wiener Polynomials of Recursively Defined Trees*
John Fink
University of Michigan–Dearborn

11:40 a.m. *The Steiner Distance Dimension of Graphs*
Michael Raines and Ping Zhang
Western Michigan University

12:00 noon *Orienting Split-Stars and Alternating Group Graphs*
Eddie Cheng and Marc J. Lipman
Oakland University

12:20 p.m. lunch on your own

SESSION 3: Alan Park, chair

2:20 p.m. *Turn Out Those Lights Now!*
Allen J. Schwenk
Western Michigan University

2:40 p.m. $K_{3,k}$ *Minors*
Thomas Bohme, **John Maharry**, and Bojan Mohar
Franklin College

3:00 p.m. *Famous Trails to Paul Erdős: Distances in the Collaboration Graph are Small*
Rodrigo De Castro and **Jerrold W. Grossman**
Universidad Nacional de Colombia and Oakland University, respectively

3:20 p.m. afternoon refreshments

SESSION 4: Curt Chipman, chair

3:50 p.m. *Distance Regular Graphs and Unimodality*
John Caughman and **Bruce Sagan**
Michigan State University

4:10 p.m. *Parameters of Bipartite Q-polynomial Distance-Regular Graphs*
John Caughman
Michigan State University

4:30 p.m. *About the Degree Matrices of Trees*
Eddie Cheng and **Marc J. Lipman**
Oakland University

≈ 5:30 p.m. survivors' dinner at King Buffet, Troy

Abstracts

(9:30 a.m.) *Geodetic Number of Random Graphs of Diameter 2*, Gab-Byung Chae, **Edgar M. Palmer**, and Wai-Cheong Siu, Michigan State University

Let S be any subset of the vertex set V of a graph G. Then the geodetic cover of S, denoted $C(S)$, consists of all vertices w such that there exist vertices u and v in S such that w lies on a geodesic between u and v. The geodetic number of a graph, denoted $\mathrm{gn}(G)$, is the cardinality of a smallest set S such that $C(S) = V$. This parameter was introduced by Harary and Buckley in their book [*Distance in graphs*, Addison-Wesley Publishing Company, 1990; **MR** 90m:05002] and has been the subject of much study. The determination of $\mathrm{gn}(G)$ was found to be NP-hard by Harary, Loukakis, and Tsouros [The geodetic number of a graph, *Graph-theoretic models in computer science, II (Las Cruces, NM, 1988–1990)*, Math. Comput. Modelling **17** (1993), no. 11, 89–95; **MR** 94d:05130]. We have found a random greedy algorithm that is very effective for a random graph $G_{n,p}$ of order n with fixed edge probability p. Our proof involves showing that $\mathrm{gn}(G_{n,p})$ is asymptotic to $\log_b(n)$, where $b = 1/(1-p)$. The methods also apply to other random graphs of diameter 2 and to random digraphs. There remains the problem of extending our results to graphs of higher diameter.

(9:50 a.m.) *How Large Can the Domination Numbers of a Graph Be?*, **Ping Zhang**, Western Michigan University

A vertex v in a graph G dominates itself as well as its neighbors. A set S of vertices in G is (1) a dominating set if every vertex of G is dominated by some vertex of S, (2) an open dominating set if every vertex of G is dominated by a vertex of S distinct from itself, and (3) a double dominating set if every vertex of G is dominated by at least two distinct vertices of S. The minimum cardinality of a set S satisfying (1), (2), and (3) is, respectively, the *domination number*, *open domination number*, and *double domination number* of G. The problem of determining the maximum value of each of these domination numbers among all graphs of a given order and size is discussed.

(10:10 a.m.) *Hypergraph Decomposition and Large Scale Optimization*, **H. Alan Park**, Oakland University

A QP (quadratic programming) problem aims to minimize a quadratic objective function under various linear constraints (consisting of equalities and inequalities). It becomes a large scale QP problem when there are a lot of variables and constraints involved in the problem. The computational demand in solving a large scale QP with traditional algorithms could be very high, and sometimes may make it virtually unsolvable. Occasionally, such a large scale problem demonstrates a certain modular structure so that we may be able to decompose the QP problem into a collection of loosely linked smaller subproblems. And then, the small subproblems can be solved independently, and the results can be coordinated to produce a global solution for the original QP. We show how the original dependency structure of the QP can be modeled by a hypergraph, and how a decomposition of the hypergraph into a collection of loosely linked subgraphs could produce a desired decomposition of the original QP. By using a hypergraph decomposition algorithm, we obtain a computationally efficient algorithm for large scale QP problems.

(11:00 a.m.) *Characterizations for the Middle of a Graph*, Paul Brown, Michigan State University, and **Garry Johns**, Saginaw Valley State University

Various notions of *distance* in graphs have been defined as a tool for studying graph structure and modeling applications. Historically, vertices of minimum distance were investigated (e.g., centers and medians). Later, vertices of maximum distance were studied (e.g., peripheries and margins). Recently, subgraphs consisting of vertices with intermediate distance have been considered (e.g., interior and annulus). In this paper, we define the *distance* of a vertex v in a connected graph G as the sum of the distances from v to every other vertex of G and focus on three distance-related subgraphs: the perfect middle, the lower middle and the upper middle. In particular, a characterization is given for each of these subgraphs.

(11:20 a.m.) *Wiener Polynomials of Recursively Defined Trees*, **John Fink**, University of Michigan–Dearborn

The *Wiener polynomial* of a connected graph G is the polynomial $W(G;q) = \sum q^{d(u,v)}$, where the sum is taken over all unordered pairs $\{u, v\}$ of distinct vertices in G, and $d(u, v)$ is the distance between u and v. Thus $W(G;q)$ is the generating function for the distance distribution $dd(G) = D_1, D_2, \ldots, D_t$, where D_i is the number of unordered pairs of distinct vertices at distance i from one another and t is the diameter of G. The derivative $W'(G;1)$ is the much-studied *Wiener index*, $W(G)$. If r is a specified vertex of a connected graph G, then the *Wiener polynomial of G relative to r* is the polynomial $W_r(G;q) = \sum q^{d(r,v)}$, where the sum is taken over all vertices v in G, including $v = r$. We discuss methods for determining the Wiener polynomial of recursively defined trees, and, as an illustration, will derive the Wiener polynomial for full k-ary trees of a given depth.

(11:40 a.m.) *The Steiner Distance Dimension of Graphs*, **Michael Raines** and Ping Zhang, Western Michigan University

For a nonempty set S of vertices of a connected graph G, the Steiner distance $d(S)$ of S is the minimum size among all connected subgraphs whose vertex set contains S. For an ordered set $W = \{w_1, w_2, \ldots, w_k\}$ of vertices in a connected graph G and a vertex v of G, the Steiner representation $s(v \mid W)$ of v with respect to W is the $(2^k - 1)$-vector

$$s(v \mid W) = (d_1(v), d_2(v), \ldots, d_k(v), d_{1,2}(v), d_{1,3}(v), \ldots, d_{1,2,\ldots,k}(v)),$$

where $d_{i_1,i_2,\ldots,i_j}(v)$ is the Steiner distance $d(\{v, w_{i_1}, w_{i_2}, \ldots, w_{i_j}\})$. The set W is a *Steiner resolving set* for G if, for every pair u, v of distinct vertices of G, u and v have distinct representations. A Steiner resolving set containing a minimum number of vertices is called a *Steiner basis* for G, and the cardinality of a Steiner basis is the *Steiner distance dimension* of G, $\dim_S(G)$. In this talk, we present some results on the Steiner dimensions of several classes of graphs.

(12:00 noon) *Orienting Split-Stars and Alternating Group Graphs*, **Eddie Cheng** and Marc J. Lipman, Oakland University

We give simple routing algorithms for a proposed orientation of alternating group graphs and split-stars. The resulting directed graphs not only are strongly connected, but they have maximal arc-connectivity and small diameter as well.

(2:20 p.m.) *Turn Out Those Lights Now!*, **Allen J. Schwenk**, Western Michigan University

Tiger Electronics has sold two forms of the Lights Out Puzzle, a square board and a cube. We show how to use linear algebra over $GF(2)$ to solve both puzzles. We show how to identify solvable and unsolvable patterns, how to find all possible solutions, and therefore how to find the minimum length solution. We determine the number of solvable patterns, both with and without symmetry. We determine the length of the longest possible minimum solution over all solvable patterns. We debunk the numerical data on the commercial package.

(2:40 p.m.) *$K_{3,k}$ Minors*, Thomas Bohme, **John Maharry**, and Bojan Mohar, Franklin College

For every $k > 0$, there is a number N_k such that every 7-connected graph on at least N_k vertices contains a $K_{3,k}$ minor. The proof uses Robertson and Seymour's structure theory for excluding a non-planar graph. This is best possible as there is a family of 6-connected graphs none of which contain $K_{3,7}$.

(3:00 p.m.) *Famous Trails to Paul Erdős: Distances in the Collaboration Graph are Small*, Rodrigo De Castro, Universidad Nacional de Colombia, and **Jerrold W. Grossman**, Oakland University

The notion of Erdős number has floated around the mathematical research community for more than thirty years, as a way to quantify the common knowledge that mathematical and scientific research has become a very collaborative process in the twentieth century, not an activity engaged in solely by isolated individuals. We explore some (fairly short) collaboration paths that one can follow from Paul Erdős to researchers inside and outside of mathematics. In particular, we find that all the Fields medalists up through 1998 have Erdős numbers less than 6, and that over 60 Nobel prize winners in physics, chemistry, economics, and medicine have Erdős numbers less than 9. In this talk we will also have some fun updating the latest statistics from the Erdős Number Project (`http://www.oakland.edu/~grossman/erdoshp.html`).

(3:50 p.m.) *Distance Regular Graphs and Unimodality*, John Caughman and **Bruce Sagan**, Michigan State University

A graph Γ is a *distance regular graph* or *drg* if, given vertices x and y at distance h, the number of vertices at distance i from x and distance j from y depends only on i, j, and h but not on the actual vertices chosen as long as they are distance h apart. In such a graph the number, k_i, of vertices at distance i from a given vertex does not depend on the vertex chosen. (So letting $i = 1$ we see that a drg is regular.) Furthermore, these numbers form a *unimodal sequence*, meaning that for some index m we have

$$k_0 \leq k_1 \leq \cdots \leq k_m \geq k_{m+1} \geq \cdots \geq k_D\,,$$

where D is the diameter of Γ. Drg's are the special case of association schemes which satisfy the P-polynomial property. We will show that similar inequalities hold in the dual Q-polynomial case under appropriate conditions. All terms will be defined in full during the talk.

(4:10 p.m.) *Parameters of Bipartite Q-polynomial Distance-Regular Graphs*, **John Caughman**, Michigan State University

Let Γ denote a bipartite distance-regular graph with diameter $D \geq 3$ and valency $k \geq 3$. Suppose that $\theta_0, \theta_1, \ldots, \theta_D$ is a Q-polynomial ordering of the eigenvalues of Γ. Leonard showed that this sequence satisfies the recurrence $\theta_{i-1} - \beta\theta_i + \theta_{i+1} = 0$ $(0 < i < D)$, for some real scalar β. Bannai and Ito conjectured that the scalar q is also real, where $\beta = q + q^{-1}$. In this talk, we show that q is real if $D \geq 4$. Moreover, if $D = 3$, then q is real unless Γ is one of the following: (i) the Heawood graph; (ii) the distance-three graph of the Heawood graph; (iii) the incidence graph of the (unique) 2-$(11, 5, 2)$ design; or (iv) a generalized hexagon of order $(1, k-1)$ for some integer k $(4 \leq k \leq 7)$.

(4:30 p.m.) *About the Degree Matrices of Trees*, Eddie Cheng and **Marc J. Lipman**, Oakland University

The degree matrix $D = (d_{i,j})$ of a graph is a generalization of the degree sequence. Here $d_{i,j}$ is defined to be the number of i-sets S with $|\mathcal{N}(S)| = j$ where

$$\mathcal{N}(S) = \{v \in V \mid (u, v) \in E \text{ for some } u \in S\}\,.$$

This is a computationally unfriendly object. But it is interesting (maybe) to ask what information about the graph its degree matrix contains. We conjecture that the degree matrix determines trees.