The maximum size of a cut and graph homomorphisms

Ju Zhou
West Virginia University

Abstract

Let $k, l > 0$ be integers. A *k-cut l-cover* of a graph H is a collection $\mathcal{F} = \{D_1, D_2, \cdots, D_k\}$ of edge cuts of H such that every edge of H lies in exactly l members of $\mathcal{F}$. For a graph G, $b(G)$ denotes the maximum size of an edge cut in G. We show that if G and H are graphs such that H has a k-cut l-cover, and that there is a graph homomorphism from G to H, then $b(G) \geq \frac{l}{k}|E(G)|$. When $p \geq 1$ and $H = C_{2p+1}$, we have $b(G) \geq \frac{2p}{2p+1}|E(G)|$ and this bound is best possible. When H is a complete graph, a former result of Erdös in 1979 is implied and furthermore, we prove the bound in it is also best possible.