Hamiltonian graphs involving neighborhood conditions

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Abstract

Let G be a graph on n vertices, δ and α be the minimum degree and independence number of G, respectively. In this paper we prove that if G is a 2-connected graph and $|N(x) \cup N(y)| \geq n - \delta - 1$ for each pair of nonadjacent vertices x, y with $1 \leq |N(x) \cap N(y)| \leq \alpha - 1$, then G is hamiltonian or $G \in \{G^*_{\frac{n-1}{2}}, K_{\frac{n+1}{2}}, K_2^* \vee 3K_{\frac{n-2}{3}}\}$ where K_2^* and $G^*_{\frac{n-1}{2}}$ are subgraphs on 2 and $\frac{n-1}{2}$ vertices respectively. As a corollary, if G is a 2-connected graph and $|N(x) \cup N(y)| \geq n - \delta$ for each pair of nonadjacent vertices x, y with $1 \leq |N(x) \cap N(y)| \leq \alpha - 1$, then G is hamiltonian. It extends the following two theorems by Faudree et al and Yin, respectively.

If G is 2-connected graph and $|N(u) \cup N(v)| \geq n - \delta$ for each pair of nonadjacent vertices $u, v \in V(G)$, then G is hamiltonian.

If G is 2-connected graph and $|N(u) \cup N(v)| \geq n - \delta$ for each pair of nonadjacent vertices u, v with $d(u, v) = 2$, then G is hamiltonian.