# Parity Edge-Coloring of Graphs

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**Abstract**

A sequence of colors $a_1, a_2, \ldots, a_{2n}$ is *repetitive* if $n > 0$ and $a_i = a_{i+n}$ for all $1 \leq i \leq n$. A *Thue coloring* of a graph $G$ colors the edges of $G$ so that no path produces a repetitive color sequence, and the *Thue number* $t(G)$ of a graph $G$ is the fewest number of colors needed for a Thue coloring. A 1906 theorem due to Thue shows that $t(P_n) = 3$ for all $n \geq 5$, and Alon, Grytczuk, Hałuszczak, and Riordan present a simple Thue coloring of the clique to show that $t(K_n) \leq 2n - 3$. In fact, this edge-coloring enjoys stronger properties.

A sequence of colors is a *parity sequence* if each color appears an even number of times. A *strong parity edge-coloring* of a graph $G$ colors the edges of $G$ so that the only walks that produce parity color sequences start and end at the same vertex, and the *strong parity edge chromatic number* $\hat{p}(G)$ of a graph $G$ is the fewest number of colors needed for a strong parity edge-coloring. The Thue coloring of $K_n$ due to Alon et. al. is a strong parity edge-coloring.

We determine $\hat{p}(K_n)$ for all $n$ and describe the strong parity edge-colorings of $K_n$ which use the minimum possible number of colors. As a corollary, we obtain a special case of a theorem due to Yuzvinsky, and we offer a conjecture on the value of $\hat{p}(K_{n,n})$ which would imply the full theorem.