

Some Recent Progresses on Hamiltonian Line Graphs and Claw-free Graphs

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Abstract

We shall present a summary of recent results we have done in this area, and methods we have developed in working on these problems. In particular, we have proved the following results.

1. (Conjectured by Ryjacek in 1990) Every 3-connected, locally N_2 -connected claw-free graph is hamiltonian.
2. (Conjectured by Kuipers and Veldman in 1996) If H is a 3-connected claw-free graph with sufficiently large order n , and if $\delta(H) \geq \frac{n+5}{10}$, then either H is hamiltonian, or $\delta(H) = \frac{n+5}{10}$ and H can be constructed from the Petersen graph.
3. Every 3-connected, essentially 11-connected line graph is hamiltonian.
4. (Open problem posed by Broersma and Veldman in 1981, JGT) Let k and s be positive integers such that $0 \leq s \leq \max\{2k, 6k - 16\}$, and let G be a k -triangular simple graph. Then $L(G)$ is s -hamiltonian if and only if $L(G)$ is $(s + 2)$ -connected.
5. Every 3-connected, $\{K_{1,3}, Z_8\}$ -free graph is hamiltonian.