List-coloring the Square of a Subcubic Graph

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Abstract

The *square* G^2 of a graph G is the graph with the same vertex set as G and with two vertices adjacent if their distance in G is at most 2. Thomassen showed that for a planar graph G with maximum degree $\Delta(G) = 3$ we have $\chi(G^2) \leq 7$. Kostochka and Woodall conjectured that for every graph, the chromatic number of G^2 equals the list-chromatic number of G^2, that is $\chi_l(G^2) = \chi(G^2)$ for all G. If true, this conjecture (together with Thomassen's result) implies that every planar graph G with $\Delta(G) = 3$ satisfies $\chi_l(G^2) \leq 7$. We prove that every planar graph with $\Delta(G) = 3$ satisfies $\chi_l(G^2) \leq 8$. In addition, we show that if G is a planar graph with $\Delta(G) = 3$ and girth $g(G) \geq 7$, then $\chi_l(G^2) \leq 7$. Dvořák, Škrekovski, and Tancer showed that if G is a planar graph with $\Delta(G) = 3$ and girth $g(G) \geq 10$ then $\chi_l(G^2) \leq 6$. We improve the girth bound to show that: if G is a planar graph with $\Delta(G) = 3$ and $g(G) \geq 9$, then $\chi_l(G^2) \leq 6$.