MIGHTY XLI Schedule

Friday evening

7:00-8:00	Reception and early registration at Sleep Inn *193 Chaffin Pl., Murfreesboro, TN*

Saturday

7:30-8:00	Registration in BAS lobby
8:00-8:10	**Tom Cheatham**, Dean of Basic and Applied Sciences Welcoming remarks, BAS S126

Sessions I-II		
	Session I: BAS S126	Session II: BAS S128
8:15-8:35	**D. Biebighauser** Prism hamiltonicity of triangulations	**P. Johnson** Are there any nontrivial uniformly $(3, r)$-regular graphs?
8:40-9:00	**B. Wei** Cycles containing ordered vertex sets	**A. Khodkar** On $(2, r)$-regular graphs
9:05-9:25	**Y. Chen** Hamilton cycles in vertex transitive graphs of order p^4	**M. Atici** New upper bounds for the integrity of cubic graphs
9:25-9:45	Break	

Plenary talk, BAS S126	
9:45-10:45	**Robin Thomas** Large 6-connected graphs with no K_6 minor

Sessions III-IV		
	Session III: BAS S126	Session IV: BAS S128
10:50-11:10	**S. Clark** Multidecompositions with the graph-pair of order four	**G. Yu** A poset problem of H-linked graphs
11:15-11:35	**M. Walsh** Some notes on lattice-ordered graphs	**Y. Shao** Edge-connectivity and edge-disjoint spanning trees
11:40-12:00	**J. Reid** Clones in matroids	**W. Cao** Decomposition of complete bipartite graphs
12:00-1:30	Lunch	

Sessions V-VI		
	Session V: BAS S126	Session VI: BAS S128
1:30-1:50	**S. Butler** Relating eigenvalues and discrepancies of graphs	**D. Slilaty** Flow and coloring duality
1:55-2:15	**R. Elsaesser** Radio Communication in random graphs	**R. Xu** On integer flows of bidirected graphs
2:20-2:40	**D. Parker** Convex invariants in multipartite tournaments	**J. Zhou** Note on group connectivity
2:45-3:00	Break	

Sessions VII-VIII		
	Session VII: BAS S126	Session VIII: BAS S128
3:00-3:20	**P. Slater** Competition-reachability of a graph	**T. Zhang** Excluded minors of nowhere-zero four-flows
3:25-3:45	**B. Stodolsky** On domination in connected cubic graphs	**Y. Zhao** Vizing's planar graph conjecture and its related problems
3:50-4:10	**Y. Liu** An overview of combinatorial methods for the haplotyping problem	**Z. Song** Some remarks on the odd case of Hadwiger's conjecture
4:15-4:40	**S. Oum** Approximating rank-width and clique-width quickly	**H. Kaul** New results in graph packing

8:00-8:10 BAS S126

Welcoming Remarks

Tom Cheatham
Dean, College of Basic and Applied Sciences
Middle Tennessee State University

8:15-8:35 BAS S126

Prism-hamiltonicity of triangulations

Daniel P. Biebighauser* — *Vanderbilt University*

M. N. Ellingham — *Vanderbilt University*

The *prism* over a graph G is the Cartesian product $G \,\square\, K_2$ of G with the complete graph K_2. If the prism over G is hamiltonian, we say that G is *prism-hamiltonian*. Prism-hamiltonicity is an interesting generalization of hamiltonicity, because the property of having a hamiltonian prism is stronger than that of having a 2-walk and weaker than that of having a hamilton path. We prove that triangulations of the plane, projective plane, torus, and Klein bottle are prism-hamiltonian. We additionally show that every 4-connected triangulation of a surface with sufficiently large representativity is prism-hamiltonian.

Are there any nontrivial uniformly $(3, r)$-regular graphs?

Peter Johnson* **Kevin Lin**
Auburn University

Caleb Petrie

A graph will be called uniformly (t, r)-regular if t is no greater than the number of its vertices, and the open neighborhood of every set of t of its vertices has cardinality r. There are various "trivially" uniformly (t, r)-regular graphs. In the case $t = 3$ these are:

(i) any empty graph on 3 or more vertices ($r = 0$);

(ii) two or more independent edges ($r = 3$);

(iii) a clique on $r > 1$ vertices, plus an isolated vertex; and

(iv) a clique on $r > 2$ vertices, minus some independent edges (where "some" includes the possibility "none").

We don't know whether or not there are any uniformly $(3, r)$-regular graphs besides these, but we have a lot of evidence that there are not.

8:40-9:00 BAS S126

Cycles containing ordered vertex sets

Bing Wei
University of Mississippi

A graph G of order n is said to be k-ordered if for every ordered sequence of k vertices, $(k \leq n)$, G contains a cycle C that encounters the sequence in the given order. In this talk, we will present some recent results on long cycles containing k vertices with a given order in a k-ordered graph. Related results on long paths will also be given.

8:40-9:00 BAS S128

On $(2, r)$-regular graphs

Abdollah Khodkar
University of West Georgia

Let G be a simple graph. G is $(2, r)$**-regular** if for any pair of distinct vertices u and w, $|N(u) \cup N(w)| = r$. G is **strongly regular** if it is regular and the number of vertices adjacent to two vertices u and w depends only on whether u and w are adjacent or not.

In this talk we prove that every $(2, r)$-regular graph is strongly regular. We also show that a $(2, r)$-regular graph of order n exists **only if** $4(n-1)(n-r)+1$ is a square.

9:05-9:25 BAS S126

Hamilton cycles in vertex transitive graphs of order p^4

Yuqing Chen
Wright State University

In this talk we present a proof that there are Hamilton cycles in vertex transitive graphs and vertex transitive digraphs of order p^4 for any prime p.

9:05-9:25 BAS S128

New upper bounds for the integrity of cubic graphs

Mustafa Atici
Western Kentucky University

Integrity, a measure of network to reliability, is defined as

$$I(G) = \min_{S \subset V} \{|S| + m(G - S)\},$$

where G is a graph with vertex set V and $m(G-S)$ denotes the order of the largest component of $G - S$. Let $|V| = n$. It is known that $\frac{n}{3} + O(\sqrt{n})$ is a general upper bound for the integrity of any cubic graph. In this article several theorems are shown that improve this general upper bound. For some families of cubic graphs an upper bound for the integrity of $\frac{n}{4} + O(\sqrt{n})$ can be established using these theorems.

Large 6-connected graphs with no K_6 minor

Robin Thomas
Georgia Institute of Technology

A graph G has an H minor if a graph isomorphic to H can be obtained from a subgraph of G by contracting edges. Jorgensen conjectured that every 6-connected graph G with no K_6 minor is apex; that is, has a vertex v such that $G \backslash v$ is planar. This is of interest, because it implies Hadwiger's conjecture for K_6-free graphs (which is known to be true, but Jorgensen's conjecture would give more structural information).

We prove that there is an absolute constant N such that Jorgensen's conjecture holds for every graph on N or more vertices. The proof uses several ingredients of independent interest: minimal non-planar extensions of planar graphs, bounded tree-width methods, and two new results about "societies," graphs with a circular order specified on a subset of their vertices.

This is joint work with Matt DeVos, Rajneesh Hegde, Kenichi Kawarabayashi, Serguei Norine and Paul Wollan.

Multidecompositions with the graph-pair of order four

Sally Clark
Birmingham-Southern College

A *graph-pair of order* t is a pair of non-isomorphic graphs G and H on t non-isolated vertices, such that $G \cup H \cong K_t$, for $t \geq 4$. Given a graph-pair (G, H), a $(G, H) -$ *multidecomposition* of a graph J is a decomposition of J into copies of G and H, including at least one copy of each. In this paper, we give constructions for (G, H)-multidecompositions of $K_n - L$ for certain leaves L, where (G, H) is the unique graph-pair on 4 vertices.

A poset problem of H-linked graphs

Gexin Yu
University of Illinois, Urbana

Given a fixed multigraph H with $V(H) = \{h_1, \dots, h_m\}$, we say that a graph G is H-linked if for every choice of m vertices $v_1, \dots, v_m$ in G, there exists a subdivision of H in G such that v_i is the branch vertex representing h_i for every i. For example, when H is a star with k edges, H-linked is equivalent to k-connected. Also, when H is a matching with k edges, H-linked is equivalent to k-linked.

Several papers have studied sufficient degree conditions (Dirac-type or Ore-type) for a graph to be H-linked (for particular H), while little attention has yet been paid to the relations between being H_1-linked and being H_2-linked for two different graphs H_1 and H_2. We consider the question of when being H_1-linked implies being H_2-linked and when it does not. We prove the following results.

- Let H_1 and H_2 be the matching and the star with k edges, respectively. Let H be a graph with k edges. We determine when H_1-linked (k-linked) implies H-linked and when H-linked implies H_2-linked (k-connected).

- Apart from a subgraph of a star, if two graphs H_1 and H_2 have the same number of vertices, then being H_1-linked implies being H_2-linked if and only if H_1 is a subgraph of H_2.

- When H_1 has more vertices than H_2, we prove sufficient conditions for H_1-linked not implying H_2-linked (and H_2-linked never implies H_1-linked).

This is joint work with Q. Liu and Douglas B. West.

11:15-11:35 BAS S126

Some notes on lattice-ordered graphs

C. D. Leach
University of West Georgia

Matthew Walsh*
Indiana-Purdue University, Fort Wayne

Consider the set of non-empty, unlabelled induced subgraphs of a graph partially ordered by inclusion. A graph is *lattice-ordered* if this poset forms a lattice; we characterize the class of lattice-ordered graphs and determine when two such graphs can have isomorphic lattices. (Joint work with C. D. Leach, University of West Georgia.)

11:15-11:35 BAS S128

Edge-connectivity and edge-disjoint spanning trees

Paul A. Catlin[1]

Hong-Jian Lai
West Virginia University

Yehong Shao*
Ohio University Southern

Given a graph G, for an integer $c \in \{2, \cdots, |V(G)|\}$, define $\lambda_c(G) = \min\{|X| : X \subseteq E(G), \omega(G-X) \geq c\}$. For a graph G and for an integer $c = 1, 2, \cdots, |V(G)|-1$, define,

$$\tau_c(G) = \min_{X \subseteq E(G) \text{ and } \omega(G-X)>c} \frac{|X|}{\omega(G-X)-c},$$

where the minimum is taken over all subsets X of $E(G)$ such that $\omega(G-X)-c > 0$. In this paper, we establish a relationship between $\lambda_c(G)$ and $\tau_{c-1}(G)$, which gives a characterization of the edge-connectivity of a graph G in terms of the spanning tree packing number of subgraphs of G. The digraph analogue is also obtained.

[1] Sadly, this author passed away on April 20, 1995.

11:40-12:00 BAS S126

Clones in matroids

Carla Cotwright
University of Mississippi

Talmage James Reid*
University of Mississippi

Jakayla Robbins
University of Montana

A pair of distinct elements in a matroid are called clones if the map that interchanges the two elements and fixes all other elements of the matroid is an automorphism of the matroid. Examples of clones include pairs of elements in a parallel class or series class of the matroid. Properties of clones are important in the study of matroid representability. We give some recent progress on studying connections between clones and minors of matroids. This is joint work with Carla Cotwright and Jakayla Robbins.

11:40-12:00 BAS S128

Decomposition of complete bipartite graphs

Weiting Cao
University of Illinois, Urbana

Suppose $K_{s,t}$ is a complete bipartite graph with $t \leq s$. We know that there exist integers n, l such that $1 + 2 + 3 + \ldots + n + l = st$. We wonder if $K_{s,t}$ can be decomposed into paths of lengths $1, 2, 3, \ldots, n$ and l. It is necessary that $n \leq 2t$ because the maximum length of a path in $K_{s,t}$ is at most $2t$. Here we prove that this condition is also sufficient. This is a generalized version of the Fink and Straight conjecture.

1:30-1:50 BAS S126

Relating eigenvalues and discrepancies of graphs

Steven Butler
University of California, San Diego

Consider the following two parameters for a graph: $\bar{\lambda}$ (a measurement of the spread of the nontrivial eigenvalues of the normalized Laplacian), and discrepancy (a measurement of how randomly the edges are distributed). It is well known that the discrepancy of a graph is bounded by $\bar{\lambda}$. Recently it was shown that for regular graphs that $\bar{\lambda}$ could be bounded in terms of discrepancy.

We will generalize discrepancy and give an explicit bound for $\bar{\lambda}$ in terms of discrepancy for all graphs with no isolated vertices. This will show that discrepancy and $\bar{\lambda}$ are equivalent in the sense that if for a family of graphs one of $\bar{\lambda}$ or discrepancy goes to zero then both go to zero.

1:30-1:50 BAS S128

Flow and coloring duality

Daniel Slilaty
Wright State University

A nowhere-zero k-flow of a directed graph G is an integer assignment to the edges from $\{-k, , -1, 1, , k\}$ such that at each vertex the inflow equals the outflow. W.T. Tutte proved that a directed planar graph G has a nowhere-zero k-flow iff its topological dual graph has a proper k-coloring. We will state and prove an analogous result for the projective plane that uses the concept of signed colorings of signed graphs.

Radio Communication in random graphs

Robert Elsaesser
University of California, San Diego

One of the most frequently studied problems in the context of information dissemination in communication networks is the broadcasting problem. We propose here several time efficient, centralized as well as fully distributed procedures for the broadcasting problem in random radio networks. In particular we show how to perform a centralized broadcast in a random graph $G_p = (V, E)$ of size $n = |V|$ and expected average degree $d = pn$ in time $O(\ln n / \ln d + \ln d)$. Later we present a randomized distributed broadcasting algorithm with the running time $O(\ln n)$. In both cases we show that the presented algorithms are asymptotically optimal by deriving lower bounds on the complexity of radio broadcasting in random graphs. In our proofs we determine some structural properties of random graphs and develop new techniques which might be useful for further research in this field. We should note here that the results of this paper hold with probability $1 - o(1/n)$.

1:55-2:15 BAS S128

On integer flows of bidirected graphs

Rui Xu*
University of West Georgia

C. Q. Zhang
West Virginia University

Bouchet conjectured that every bidirected graph which admits a nowhere-zero integer flow admits a nowhere-zero 6-flow. We prove that Bouchet's conjecture is true for 6-edge connected graphs.

2:20-2:40 BAS S126

Convex invariants in multipartite tournaments

Darren Parker
University of Dayton

Let T be a convexity space. Among the most studied convex invariants are the Helly number $h(T)$ and the Radon number $r(T)$. Another relatively well-known invariant is the rank $d(T)$. It is well-known that $h(T) \leq r(T) \leq d(T)$. We study these invariants in the case where $T = (V, E)$ is a multipartite tournament under two-path convexity. In this case, a subset $C \subseteq V$ is convex if, whenever $u, v \in C$ and $w \in V$ with $u \to w \to v$, then $w \in C$. Our aim is to determine the circumstances under which $h(T) = r(T) = d(T)$. We give a partial answer to this question. Along the way, we prove a result relating the structure of a multipartite tournament to its convex independent sets.

2:20-2:40 BAS S128

Note on group connectivity

Ju Zhou
West Virginia University

It is conjectured that every 4-edge connected graph with each edge contained in a circuit of length at most 3 admits a nowhere-zero Z_3-flow. Devos made a stronger conjecture by stating that every such graph is Z_3-connected. In this paper, we get a family of graphs which are the counterexamples to the stronger conjecture. Moreover, motivated by the counterexamples, we prove that every essential 6-edge connected graph with at most one degree 4 vertex and whose edge set is an edge disjoint union of circuits of length at most 3 is Z_3-connected. In particular, every 6-edge connected such graph is Z_3-connected.

3:00-3:20 BAS S126

Competition-reachability of a graph

Suk Jai Seo
Middle Tennessee State University

Peter J. Slater*
University of Alabama, Huntsville

The reachability $r(D)$ of a directed graph D is the number of ordered pairs of distinct vertices (x, y) with a directed path from x to y. Consider a game associated with a graph $G = (V, E)$ involving two players (maximizer and minimizer) who alternately select edges and orient them. The maximizer attempts to maximize the reachability, while the minimizer attempts to minimize the reachability, of the resulting digraph. If both players play optimally, then the reachability is fixed. Parameters that assign a value to each graph in this manner are called competitive parameters. We discuss bounds and also determine the competitive-reachability for special classes of graphs

3:30-3:50 BAS S128

Excluded minors of nowhere-zero four-flows

Taoye Zhang
West Virginia University

Tutte conjectured theat every bridgeless graph without a P_{10}-minor admits a nowhere-zero 4-flow. It's proved in this talk that every bridgeless graph without a minor isomorphic to one of a list of graphs admits a nowhere-zero 4-flow. As a corollary, we prove every bridgeless graph without a P_{10}/e-minor admits a nowhere-zero 4-flow.

3:25-3:45 BAS S126

On domination in connected cubic graphs

A. Kostochka
University of Illinois, Urbana

B. Y. Stodolsky*
University of Illinois, Urbana

In 1996, Reed proved the domination number $\gamma(G)$ of every n-vertex graph G with minimum degree at least three is at most $3n/8$. Also, he conjectured that $\gamma(H) \leq \lceil n/3 \rceil$ for every connected 3-regular (cubic) n-vertex graph H. In this note, we disprove this conjecture. We construct a connected cubic graph G on 60 vertices with $\gamma(G) = 21$ and present a sequence $\{G_k\}_{k=1}^{\infty}$ of cubic graphs with

$$\lim_{k\to\infty} \frac{\gamma(G_k}{|V(G_k)|} \geq 8/23 = 1/3 + 1/69.$$

3:25-3:45 BAS S128

Vizing's planar graph conjecture and its related problems

Yue Zhao
University of Central Florida

Let G be a graph with maximum degree Δ and S be a surface. We define: $\Delta(S) = \max\ \{\Delta(G)|\ G$ is a class two graph embedded in S $\}$. Hence Vizing's planar graph conjecture can be restated as: If S is the plane, then $\Delta(S) = 5$. In this talk, we will discuss $\Delta(S)$ for some surfaces.

An overview of combinatorial methods for the haplotyping problem

Yunkai Liu
University of South Dakota

The investigation of genetic differences among humans has given evidence that mutations in DNA sequences are responsible for some genetic diseases. Single nucleotide polymorphisms (SNPs) are the most frequent forms of human genetic variations. A complete map of all SNPs occurring in the human populations will be extremely valuable for studying specific haplotypes with specific genetic diseases. To construct such a map, determining the DNA sequences that form all chromosomes is required. In diploid organisms like human, each chromosome consists of two sequences called *haplotypes*. Distinguishing the information contained in both haplotypes when analyzing chromosome sequences poses several new computational issues which collectively form a new merging topic of Computational Biology known as *Haplotyping*. The recent discovery that genomic DNA can be partitioned into long blocks where genetic recombination has been rare, makes related mathematical research meaningful and leads more computational challenges.

In this talk, the haplotype inference problem, the pure parsimony problem and the prefect phylogeny problem will be introduced. And several new combinatorial approaches to those problems will be reviewed.

Some remarks on the odd case of Hadwiger's conjecture

Ken-ichi Kawarabayashi
Tohoku University

Zi-Xia Song*
University of Central Florida

We say that H has an *odd complete minor* of order at least k if there are k vertex disjoint trees in H such that every two of them are joined by an edge, and in addition, all the vertices of trees are two-colored in such a way that the edges within the trees are bichromatic, but the edges between trees are monochromatic.

Gerards and Seymour conjectured that if a graph has no odd complete minor of order k, then it is $(k-1)$-colorable. This is an analogue of the well-known conjecture of Hadwiger. Recently, Geelen et al. proved that there exists a constant c such that any graph with no odd K_k-minor is $ck\sqrt{\log k}$-colorable. But, it is not known if there exists an absolute constant c such that any graph with no K_k-minor is ck-colorable nor if any graph with no odd K_k-minor is ck-colorable.

Motivated by these facts, in this paper, we shall first prove that, for any k, there exists a constant $f(k)$ such that any $(496k+13)$-connected graph with at least $f(k)$ vertices has either an odd K_k-minor or a vertex set X of order at most $8k$ such that $G-X$ is bipartite. We also prove that every graph on n vertices has an odd complete minor $K_{\lceil n/2\alpha(G)-1\rceil}$, where $\alpha(G)$ denotes the independence number of G. This is an analogous result of Duchet and Meyniel. We prove a slightly stronger result when $\alpha(G)=3$.

This is joint work with Ken-ichi Kawarabayashi.

Approximating rank-width and clique-width quickly

Sang-il Oum
Georgia Institute of Technology

Rank-width is defined by Seymour and the author to investigate clique-width; they show that graphs have bounded rank-width if and only if they have bounded clique-width. It is known that many hard graph problems have polynomial-time algorithms for graphs of bounded clique-width, however, requiring a given decomposition corresponding to clique-width (*k-expression*); they remove this requirement by constructing an algorithm that either outputs a rank-decomposition of width at most $f(k)$ for some function f or confirms rank-width is larger than k in $O(|V|^9 \log |V|)$ time for an input graph $G = (V, E)$ and a fixed k. This can be reformulated in terms of clique-width as an algorithm that either outputs a $(2^{1+f(k)} - 1)$-expression or confirms clique-width is larger than k in $O(|V|^9 \log |V|)$ time for fixed k.

We develop two separate algorithms of this kind with faster running time. We construct a $O(|V|^4)$-time algorithm with $f(k) = 3k + 1$ by constructing a subroutine for the previous algorithm; we may now avoid using general submodular function minimization algorithms used by Seymour and the author. Another one is a $O(|V|^3)$-time algorithm with $f(k) = 24k$ by giving a reduction from graphs to binary matroids; then we use an approximation algorithm for matroid branch-width by Hliněný.

New results in graph packing

Hemanshu Kaul*
University of Illinois, Urbana

A. Kostochka
University of Illinois, Urbana

Let G_1 and G_2 be graphs of order at most n, with maximum degree Δ_1 and Δ_2, respectively. We say that G_1 and G_2 pack if their vertex sets map injectively into $[n]$ so that the images of the edge sets are disjoint. Note that the concept of graph packing generalizes various extremal graph problems, including problems on forbidden subgraphs, fixed subgraphs and equitable coloring. The study of packings of graphs was started in the 1970s by Sauer and Spencer and by Bollobás and Eldridge.

Sauer and Spencer showed that if $\Delta_1\Delta_2 < \frac{n}{2}$, then G_1 and G_2 pack. We extend this by characterizing the extremal graphs : if $\Delta_1\Delta_2 \leq \frac{n}{2}$, then G_1 and G_2 fail to pack if and only if one of G_1 or G_2 is a perfect matching and the other either is $K_{\frac{n}{2},\frac{n}{2}}$ with $\frac{n}{2}$ odd or contains $K_{\frac{n}{2}+1}$. This result can thought of as small step towards the well-known Bollobás-Eldridge graph packing conjecture. We will also mention other new work related to this conjecture.

This is joint work with Alexandr Kostochka.