

New methods for orthogonal double covers of complete graphs by trees

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An *orthogonal double cover* of the complete graph K_n by a graph G is the set of n subgraphs G_1, G_2, \dots, G_n of K_n with the following properties:

- (1) G has $n - 1$ edges and $G_i \cong G$ for every $i = 1, 2, \dots, n$;
- (2) every edge of K_n appears in exactly two copies of G (double cover property);
- (3) every two distinct copies G_i, G_j of G intersect in exactly one edge (orthogonality property).

Gronau, Mullin, and Rosa conjectured that for every tree T with n vertices except for P_4 there exists an ODC of K_n by T . They also proved the conjecture for all caterpillars of diameter 3. Later, Leck and Leck proved it for all caterpillars of diameter 4 and all trees with up to 14 vertices. We prove the conjecture for all caterpillars of diameter 5 and order $n \geq 24$; for orders $15 \leq n \leq 23$ we prove it with several exceptions, which we believe are only temporary.

The method we use is a common generalization of methods developed for ODCs by Gronau, Mullin, and Rosa and by Leck and Leck and for complete graph factorizations by Tereza Kovarova.

Key words: Orthogonal double cover, orthogonal labeling, fixing labeling.

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