

ON THE INDEPENDENCE NUMBERS OF THE
POLARITY GRAPHS OF THE CLASSICAL
PROJECTIVE PLANE

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ABSTRACT. The classical projective plane $PG(2, q)$ is known to admit only two polarity graphs: the Erdős-Rényi Graph ER_q which is well-known for its extremal properties, and the lesser known Unitary Polarity Graph U_q which exists only for square prime powers q . In this talk we will discuss the independence number of each graph.

It will be shown for q sufficiently large that $.19239q^{3/2} \leq \alpha(ER_q) \leq q^{3/2}$ resolving the question of the magnitude of $\alpha(ER_q)$. Better lower bounds will be given for specific q ; in particular it will be shown that for even powers of two $\alpha(ER_q) = q^{3/2} + O(q)$. The lower bound is derived from explicit constructions of independent sets in ER_q , while the upper is derived from combinatorial and eigenvalue bounds.

We will also demonstrate that the independence number of the Unitary Polarity Graph of $PG(2, q)$, q a square, is precisely $q^{3/2} + 1$.

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