Improved bounds for the crossing numbers of $K_{m,n}$ and K_n

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It has long been conjectured that the crossing number $cr(K_{m,n})$ of the complete bipartite graph $K_{m,n}$ is equal to $Z(m,n) := \lfloor \frac{m-1}{2} \rfloor \lfloor \frac{m}{2} \rfloor \lfloor \frac{n-1}{2} \rfloor \lfloor \frac{n}{2} \rfloor$. Another long-standing conjecture is that the crossing number $cr(K_n)$ of the complete graph K_n is equal to $Z(n) := \frac{1}{4} \lfloor \frac{n}{2} \rfloor \lfloor \frac{n-1}{2} \rfloor \lfloor \frac{n-2}{2} \rfloor \lfloor \frac{n-3}{2} \rfloor$.

In this talk, I will outline a new method that improves the asymptotic lower bounds to $0.83Z(m,n)$ and $0.83Z(n)$ respectively. This is follows from the improved lower bound $cr(K_{7,n}) \geq 2.1796n^2 - 4.5n$. The proof uses combinatorial ideas as well as quadratic optimization techniques.

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