Group connectivity of graphs with diameter at most 2

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Abstract

Let G be an undirected graph, A be an (additive) abelian group and $A^* = A - \{0\}$. A graph G is A-connected if G has an orientation $D(G)$ such that for every function $b : V(G) \mapsto A$ satisfying $\sum_{v \in V(G)} b(v) = 0$, there is a function $f : E(G) \mapsto A^*$ such that at each vertex $v \in V(G)$, the amount of f values on the edges directed out from v minus the amount of f values on the edges directed into v equals $b(v)$. In this paper, we investigate, for a 2-edge-connected graph G with diameter at most 2, the group connectivity number $\Lambda_g(G) = \min\{n : G \text{ is } A\text{-connected for every abelian group } A \text{ with } |A| \geq n\}$, and show that any such graph G satisfies $\Lambda_g(G) \leq 6$. Furthermore, we show that if G is such a 2-edge-connected diameter 2 graph, then $\Lambda_g(G) = 6$ if and only if G is the 5-cycle; and when G is not the 5-cycle, then $\Lambda_g(G) = 5$ if and only if G is the Petersen graph or G belongs to two infinite families of well characterized graphs.

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