Differential Geometry

Fall Semester 2014

Course MATH 445 Section 1 MWF 3:00–3:50 pm RB 447

Instructor Dr. Hanspeter Fischer

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Office Hours Mon 2:00–2:50 pm, Tue 10:00–10:50 am, Wed 9:00–9:50 am, Thu 9:00–9:50 am, Fri 2:00–2:50 pm, and by appointment.

Prerequisites MATH 267

Text *Elementary Differential Geometry*, by Barrett O'Neill (Academic Press 2006, Revised Second Edition)

Website The above website features hints to selected homework problems and additional reading. Please visit this site regularly for course announcements and up-to-date deadlines.

Contents We begin with a brief survey of Chapter 1, which reviews some concepts of Calculus III while introducing new language and notation. We will then work through Chapters 2 and 4: the calculus of curves and surfaces in 3-space using *moving frames* and *coordinate patches*. Next, in Chapter 5, we extend the notion of *curvature*, as encountered for curves in Calculus III, to surfaces in 3-spaces. In Chapter 6, we will discover which features of the shape of a surface are *intrinsic* to the surface (i.e., can be observed from within the surface, without any reference to 3-space that surrounds it) and which are not. In Chapter 7, we will explore this intrinsic geometry of surfaces. In fact, we will learn how to perform calculus-style geometry within surfaces, which cannot even be created inside of 3-space, like the classical example of non-Euclidean geometry: the hyperbolic plane. As time permits, the course will end with a brief discussion of the important connections between local and global geometry, as well as the connections between length minimizing paths and curvature (Sections 7.6, 7.7, 8.1-8.3).

Historical Context Ever since Euclid proposed his postulates for geometry, around 300 B.C., mathematicians were debating the possibility of the Parallel Postulate being *deducible* from the other axioms on his list. The discovery of the hyperbolic plane (by Lobatchevski in 1829 and, independently, by Bolyai in 1831) put a definite halt to this endeavor, because it satisfies all postulates of Euclid, except the Parallel Postulate. The fact that this non-Euclidean geometry, in contrast to the spatial spherical geometry, was perceived without reference to 3-space—indeed, cannot even be created inside of 3-space—was initially regarded as a pathological curiosity. However, Gauss, already in 1827, recognized the potential of a concept of geometry that functions solely from within, one that requires no reference to an outside "world." He was the

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first to realize which aspects of curvature, as introduced by Euler in 1760, were of an intrinsic nature. These ideas were taken up again, refined, and generalized to higher dimensions by Riemann in 1854. In 1916, Riemannian geometry became invaluable to modern Physics, when its ideas were applied to Einstein's theory of general relativity, as it discusses the intrinsic curvature and large scale structure of our universe; the so-called 4-dimensional space-time continuum.

Homework Homework assignments will be posted as we work through the material. Some problems will be assigned for practice only, while other designated problems will be collected and graded. As a rule, homework will be due on the Friday of the following week; the exception being the week before an exam, during which other due dates might be announced. Late homework will not be accepted. The assignments (with due dates) will be available online at above website.

When submitting exercises make sure to have your name and course number on it to be eligible for credit. Please staple your solutions.

Examinations There will be two midterm examinations. The comprehensive final exam will be on Wednesday, December 10, 2:15–4:15 pm. Each exam will have a take-home component.

Evaluation Each exam, including the final exam, constitutes 25% of your grade. Homework accounts for the remaining 25%.

The approximate grading scale (in percent) is:
A: 90 – 100, B: 80 – 90, C: 70 – 80, D: 60 – 70, F: below 60.

Timetable A tentative schedule is attached. See above website for updates.

Deadlines The last day to change a course is Friday, August 22. The course withdrawal period is Saturday, August 23, through Wednesday, October 22. During this period students can elect to receive a "W" for the course by completing and submitting the proper form to the Registrar's Office. The instructor's permission is not required. For details, see the section *Withdrawal* under *Degree Requirements and Time Limits* in the current Undergraduate Catalog.

General Remarks

(1) Students are expected to come prepared to all meetings, having read upcoming sections of the textbook. Be always up to date!

(2) It takes time to digest and master new abstract concepts. Expect to study for a minimum of two hours outside of class for each hour in class. Missing class is a major setback.

(3) Everybody is strongly encouraged to engage with classmates in frequent general discussions of the material. However, on assignments, credit should be claimed only for one's own work. Moreover, *collaboration on take-home exams is not permitted.*

(4) If you need course adaptations or accommodations because of a disability, please make an appointment with me as soon as possible. Ball State's Disability Services office coordinates services for students with disabilities; documentation of a disability needs to be on file in that office before any accommodations can be provided. Disability Services can be contacted at 285-5293 or dsd@bsu.edu.

(5) Please turn cell phones off during class. Thank you.