CS 324
Analysis and Design of Algorithms

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Administrative Things

- Roster
- Syllabus
  - Topics
  - Prerequisites
    - 121 Programming
    - 124 Discrete Math
    - Math 217 Linear Algebra
  - Textbook
- Mathematical class…
  - Don't be afraid
  - Be interactive, work with me and the class
  - Come to my office, email me
  - Work consistently
  - Read the textbook, take your time with homework
  - If you have taken this class before, come see me

Algorithms

- What is an algorithm?
- Why do we care about it?
- How do we write it down?
- What can we find out about it?
- How do we invent one?

Title: Applesauce Oatmeal Cookies
Categories: Cookies
Yield: 36 servings

1 3/4 c Flour
1/2 ts Salt
1 ts Cinnamon
1/2 ts Nutmeg
1/2 ts Powdered cloves
1 c Applesauce
1 ts Baking soda
1/2 c Shortening
1 c White sugar
1 Egg
1 c Seedless raisins
1 c Rolled oats; quick cooking

Mix shortening with sugar and egg until creamy. Combine applesauce with soda. Mix well add rest of ingredients and mix well. Drop by teaspoon 2" apart on greased cookie sheet. Bake 350~ 20 minutes or until golden brown.

Examples of Algorithms

- Easter
- Euclid's algorithm
- Sieve of Eratosthenes

- Sorting
- Searching
- Hashing
- Compression
- Hardware

What are Algorithms Good for?

What are Algorithms Good for?

- Searching the Web
  - String matching
  - Hashing
- Genome databases
  - Pattern matching
  - Searching
- Protein folding
- Internet routing (TCP/IP)
- Cryptography
  - Factorization
  - Cryptographic hashes
  - Secure Web sites, passwords
- Production Scheduling
- Shortest distance on a road map
- Ray tracing

Example 1

- Add: 15367 + 16419
- Steps?
- Precise?
- Input / output?
- General?
- Repeatable?
- Fast?
- Needs space?

Topics in this Class

- O-Notation
- Greedy Algorithms
- Divide-and-Conquer
- Dynamic Programming
- Graphs
- Probabilistic Algorithms
- String Matching
- Computational Complexity

Example 2: Multiplication

- Multiply 981 with 1234
- Algorithm?
- Is there another way to do it?
  - A better way???

Multiplication à la Russe

981	1234	1234
490	2468	
245	4936	4936
122	9872	
61	19744	19744
30	39488	
15	78976	78976
7	157952	157952
3	315904	315904
1	631808	631808
		1210554

Algorithm – à la Russe

```
int russe(int m, int n) {
   int result = 0;
   do {
        if( m % 2 == 1 ) {
            result += m;
        }
        m = m / 2;
        n = n + n;
   } while( m > 1 )
   return result;
}
```

Multiplication the 3rd

- 981 * 1234

		shift	
09	12	4	108....
09	34	2	306..
81	12	2	972..
81	34	0	2754
			1210554

Multiplication the 3rd

- Divide-and-Conquer multiplication
- 9 * 12

		shift	
0	1	2	0..
0	2	1	0.
9	1	1	9.
9	2	0	18
			108

Multiplication – Summary

- Three very different ways
  - elementary school way
  - à la Russe
  - divide-and-conquer
  - a fourth one in the book…
- Advantages? Disadvantages?
  - speed?
  - space?
  - simplicity?
  - basic operations?
- Recursion versus looping?

Math Review

- set
  - empty set
  - subset
  - union
  - intersection
  - pair
  - Cartesian product
- relation
  - examples: <, =, divides
- function
  - domain
  - image
- quantifiers
- logarithm
  - base 10
  - base 2

Proofs

- direct proof
- by contradiction (reductio ad absurdum)
- by mathematical induction

- Why proofs?
  - pro: reliable
  - con: more work
  - frequently: people just test algorithm
    - problem with testing?

Direct Proof

- General schema
  - we know p is true
  - we replace terms in p by their definitions
  - we apply some transformations
  - we receive q
- Example:
  - prove: 2+2=4

  - definitions? transformations?

Proof by Contradiction

- Euclid: There are infinitely many prime numbers.

- terminology
  - infinitely?
  - prime number?

- proof?
  - by counterexample?
  - direct proof?

Proof of Euclid's Theorem

- Proof:
  - x := product of all primes
  - y := x+1
  - d: smallest divisor of y (except 1)
  - d must be prime
  - hence: d divides x
  - contradiction!!!

- Proof by contradiction
  1. assume opposite
  2. draw conclusions from it
  3. find a contradiction
     - hence: original statement must be false!

Induction, Deduction

- Induction
  - 2,4,6,8,10,12,…?
  - prime numbers: 3,5,7,…?
  - empirical physics, biology
- Deduction
  - logic, proofs
  - theoretical physics
  - math?
- Mathematical induction
  - NOT the same as induction
  - INSTEAD: a kind of deduction
  - induction basis, induction step

Mathematical Induction: Example

- To prove: $\Sigma^{n}_{k=1} k = n(n+1)/2$
  - find counterexample?
- Proof:
  - Base case: (n=1) $\Sigma^{1}_{k=1} k = 1 = 1(1+1)/2$
  - Induction hypothesis: $\Sigma^{n-1}_{k=1} = (n-1)n/2$
  - Induction Step $\Sigma^{n}_{k=1} k = \Sigma^{n-1}_{k=1} k + n$

    $= (n-1)n/2 + n$

    (substituting based on I.H.)

    $= (n-1)n/2 + 2n/2$

    $= n(n-1+2)/2$

    $= n(n+1)/2$
  - proven!

Mathematical Induction

- Usable for:
  - proving a property about natural numbers: p(n)
- Needed:
  - Induction basis
    - typically 0 or 1
    - show that the statement is true for basis
  - Induction step
    - show that p(n-1) implies p(n)
  - Hence: has to be true for all natural numbers >= basis.