

Test 1

Talking to other students and the use of notes, books, or electronic devices are not allowed and are considered cheating. There is a total of 84 points plus extra credit; but you cannot get more than 84 points.

1. (12 points) Give a short definition of the following:

a) algorithm

b) Ω

c) asymptotic notation

2. (12 points) Prove the following statement by mathematical induction.

If a positive integer is dividable by 3, then the sum of its digits is also dividable by 3.

(Example: 18 is dividable by 3 because $18=3\cdot 6$. The sum of its digits is $1+8=9$, which is also dividable by 3.)

3. (24 points) Examine the following well-known algorithm.

```
int gcd( int m, int n ) {  
    if( n==0 ) {  
        return m;  
    }  
    return gcd( n, m % n );  
}
```

a) What does “gcd” stand for?

b) What is the name of this algorithm?

c) Calculate $\text{gcd}(252, 52)$. List all intermediate calls to gcd.

d) Give another algorithm that solves the same problem. It does not have to be fast.

4. (12 points) Arrange all of the following functions in increasing order of their $O()$ -sets, using the symbols \subset and $=$.

n
 $\log n^5$
 2^n
 $n \log n$
 n^c , with $c \geq 0$

5. (12 points) What is $\Theta(f(n))$? Give a proof.

$$f(n) = 3 \log n + 25 n^2 - 345 + \sin n$$

6. (12 points) Algorithm A takes n hours to solve a given problem with n input objects. Algorithm B takes n^3 milliseconds to solve the same problem. What is the smallest value of n for which A is faster than B?

7. (18 points extra credit) Prove that $(\log n)^2 \in O(\sqrt{n})$. Hint: do not use mathematical induction.