On Irreducible No-hole $L(2,1)$-coloring of the Cartesian Product of a Path and a Tree

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An $L(2,1)$-*coloring* of a graph G is a mapping $f\colon V(G) \to Z^+ \cup \{0\}$ such that $|f(u) - f(v)| \geq 2$ for all edges uv of G, and $|f(u) - f(v)| \geq 1$ if u and v are at distance two in G. The *span of an L(2, 1)-coloring* f of G, denoted by span(f), is $\max\{f(v)\colon v \in V(G)\}$. The *span* of G, denoted by $\lambda(G)$, is the minimum span of all possible $L(2,1)$-colorings of G. If f is an $L(2,1)$-coloring of a graph G with span k then an integer l is a *hole* in f, if $l \in (0,k)$ and there is no vertex v in G such that $f(v) = l$. A *no-hole coloring* is defined to be an $L(2,1)$-coloring with no hole in it. An $L(2,1)$-coloring is said to be *irreducible* if the color of none of the vertices in the graph can be decreased and yield another $L(2,1)$-coloring of the same graph. An *irreducible no-hole coloring* of a graph G, in short *inh-coloring* of G, is an $L(2,1)$-coloring of G which is both irreducible and no-hole. A graph G is *inh-colorable* if there exists an inh-coloring of it. For an inh-colorable graph G the *lower inh-span* or simply *inh-span* of G, denoted by $\lambda_{inh}(G)$, is defined as $\lambda_{inh}(G) = \min\{\text{span}(f)\colon f \text{ is an inh-coloring of } G\}$. In this paper, we prove that the Cartesian product of a tree and a path is inh-colorable.