On the Strength of Some Trees

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Let G be a graph of order p. A numbering f of G is a labeling that assigns distinct elements of the set $\{1, 2, \ldots, p\}$ to the vertices of G, where each edge uv of G is labeled $f(u) + f(v)$. The strength $\operatorname{str}_f(G)$ of a numbering $f : V(G) \to \{1, 2, \ldots, p\}$ of G is defined by

$$\operatorname{str}_f(G) = \max\{f(u) + f(v) \,|\, uv \in E(G)\},$$

that is, $\operatorname{str}_f(G)$ is the maximum edge label of G, and the strength $\operatorname{str}(G)$ of a graph G itself is

$$\operatorname{str}(G) = \min\{\operatorname{str}_f(G) \,|\, f \text{ is a numbering of } G\}.$$

In this paper, we show that some classes of trees attains the sharp bounds. We also establish a formula for the strength of the corona of certain graph and arbitrary number of isolated vertices. Therefore, applying this result with classes of trees studied in this paper, we obtain the exact value of strength for other classes of trees.

Especially, we show the exact value of strength for every caterpillar and also for some lobsters.