Generating Expressions for a Family of Labeled Grid Graphs

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Given a directed graph $G = (V, E)$ with one source and one sink, an *edge labeling* is a function $E \longrightarrow R$, where R is a ring equipped with two binary operations $+$ (addition or disjoint union) and $\cdot$ (multiplication or concatenation). Elements of R are called *labels*, and a *labeled graph* refers to an edge-labeled graph with all labels distinct. A graph expression consists of labels, the two ring operators $+$ and $\cdot$, and parentheses. We define the total number of labels in the expression as its *complexity*.

We investigate a family of *directed grid graphs* $G_{m,n}$ having $m \times n$ vertices. Each vertex in these graphs corresponds to a unique pair of integers (i, j) $(1 \leq i \leq m,\ 1 \leq j \leq n)$ which are vertex coordinates. All graphs of the family (specifically, a *simple grid*) have edges $\{((i, j), (i + 1, j)) \mid 1 \leq i < m, 1 \leq j \leq n\} \cup \{((i, j), (i, j + 1)) \mid 1 \leq i \leq m, 1 \leq j < n\}$. A *one-diagonal grid* have also diagonal edges $\{((i, j), (i+1, j+1)) \mid 1 \leq i < m, 1 \leq j < n\}$ and a *two-diagonal grid* includes additionally secondary-diagonal edges $\{((i, j), (i - 1, j + 1)) \mid 1 < i \leq m, 1 \leq j < n\}$. In all graphs we consider m as a constant which determines the *depth* of a graph, while n characterizes the *size* of the graph. Besides, we study a *square grid graph*, $G_{n,n}$. In order to generate and to simplify the expressions of labeled grid graphs we present a *decomposition method*.

For a simple grid $G_{m,n}$ we construct expressions of quasi-linear, $O\left(n \log^{m-1} n\right)$ complexity. In addition, we show that the decomposition method being applied to a square simple grid of size $n \times n$, is able to generate expressions of quasi-polynomial complexity, namely, $O\left(n^{\lceil \log_2 n \rceil + 2}\right)$. Although lengths of expressions derived for one-diagonal grids $G_{m,n}$ are greater than for corresponding simple grids, their order is the same, $O\left(n \log^{m-1} n\right)$. However, complexities of expressions of two-diagonal grids $G_{m,n}$ significantly increase. The question is how to obtain the expressions whose complexities polynomially depend on n. To solve this problem we intend to develop and to modify accordingly the decomposition method.