A Critique of One-Tailed Hypothesis Test Procedures in Business and Economics Statistics Textbooks

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The ability to conduct and correctly interpret the results of hypothesis tests is one of the most important skills that students can acquire in the introductory statistics course. Unfortunately, it is also one of the most difficult skills for them to learn.[1] Although this problem is widely recognized, textbook authors disagree about the best approach to use for conducting one-tailed hypothesis tests.

In a recent survey of introductory business and economics statistics textbooks, we found that about half use the simple null hypothesis approach and about half use the composite null hypothesis approach for one-tailed hypothesis tests.[2] Whereas both approaches are valid, the composite null hypothesis approach requires a more detailed statistical explanation than does the simple null hypothesis approach. Unfortunately, most textbooks that use the composite null hypothesis approach eschew the relevant explanation and use, instead, an explanation suitable only for the simple null hypothesis approach.

STATISTICAL DIFFERENCES

The first step in a one-tailed hypothesis test is the choice of the appropriate null and alternative hypotheses. Under the simple null hypothesis approach, the appropriate choices for a one-tailed test of the population mean, μ, with a specified numerical value, μ_0, are either

$$\begin{cases} H_0: \mu = \mu_0 \\ H_a: \mu < \mu_0 \end{cases} \quad \text{or} \quad \begin{cases} H_0: \mu = \mu_0 \\ H_a: \mu > \mu_0 \end{cases}.$$

Using this approach, the null hypothesis, H_0, specifies the same unique value, μ_0, for the population mean regardless of the form of the alternative hypothesis, H_a.

In contrast, under the composite null hypothesis approach, the appropriate choices are either

$$\begin{cases} H_0: \mu \geq \mu_0 \\ H_a: \mu < \mu_0 \end{cases} \quad \text{or} \quad \begin{cases} H_0: \mu \leq \mu_0 \\ H_a: \mu > \mu_0 \end{cases}.$$

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With this approach, the composite null hypothesis, H_0, specifies a different range of possible values for the population mean depending upon the appropriate alternative hypothesis.

Because of this difference in the form of the null hypothesis, the two approaches require different statistical explanations. Consider, for example, a one-tailed hypothesis test about the mean of the population where the appropriate alternative hypothesis is H_a: $\mu < \mu_0$. Suppose, further, that the sample mean, $\bar{x}$, is selected as the test statistic.[3] With the simple null hypothesis approach, the sampling distribution of $\bar{x}$ under the null hypothesis is uniquely determined because the value of the population mean is uniquely specified, H_0: $\mu = \mu_0$ (Figure 1). Because this approach yields a unique sampling distribution, the probability of a Type I error is uniquely determined for *any given rejection* region. Therefore, the appropriate rejection region for the test is the one for which the probability of its Type I error equals α, the chosen significance level (shown in Figure 1).

In contrast, the sampling distribution of $\bar{x}$ under the null hypothesis is not uniquely determined using the composite null hypothesis approach because the value of the population mean is not uniquely specified. Instead, there are an infinite number of alternative sampling distributions under the composite null hypothesis H_0: $\mu \geq \mu_0$. Three of these are shown in Figure 2.[4] Because of this multiplicity of sampling distributions, the probability of a Type I error associated with *any specific* rejection region is not unique. Its value depends upon which of the many sampling distributions is being considered. However, the *largest* probability of a Type I error for any chosen rejection region is associated with the sampling distribution centered at μ_0. We thus select the rejection region that makes the probability of a Type I error equal to α for this specific sampling distribution (but less than α for all other ones specified under the null hypothesis). Under this approach, the rejection region is chosen to make the *maximum* (but not the actual) probability of a Type I error equal to α (Figure 2).[5]

The above discussion and Figures 1 and 2 indicate that both approaches are statistically valid, yield identical rejection regions, and produce identical decisions. However, the statistical explanation underlying the simple null hypothesis approach differs from that for the composite null hypothesis approach in two ways: the existence of single versus multiple sampling distributions under the null hypothesis and the exact versus maximum probability of the Type I error associated with the level of significance, α. Because of these differences, textbook authors who use the composite null hypothesis approach must provide a different and somewhat more detailed explanation of one-tailed hypothesis tests than is necessary for textbook authors who rely on the simple null hypothesis approach. The problem for students, however, is that, by and large, authors fail to do so.

MISSING EXPLANATIONS

We surveyed 44 introductory business and economics statistics textbooks to examine their coverage of one-tailed hypothesis test procedures (Table 1). Twenty books in our survey use the simple null hypothesis approach, and 24 use the

FIGURE 1
$\bar{x}$
Sampling Distribution of $\bar{x}$ Under the Simple Null Hypothesis
H_0: $\mu = \mu_0$

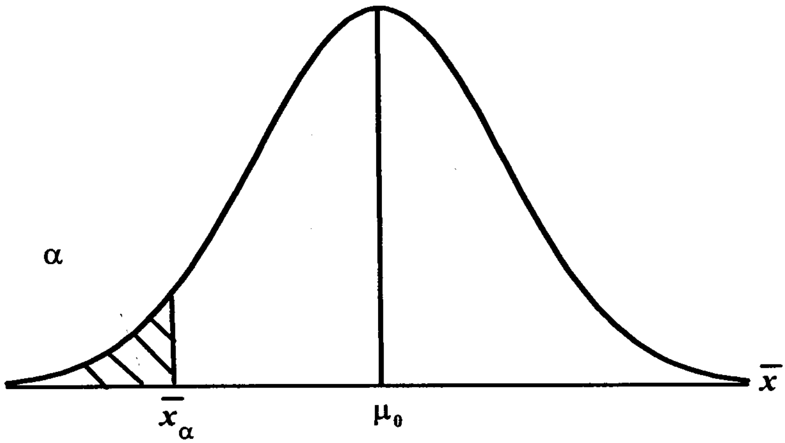
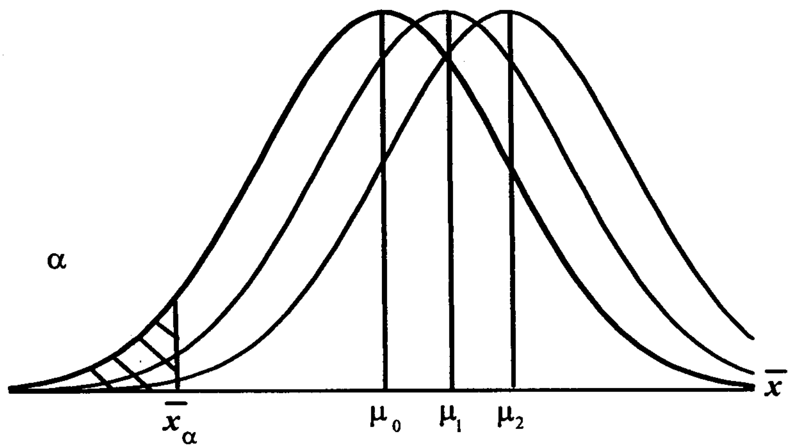


FIGURE 2
Sampling Distribution of $\bar{x}$ Under The Composite Null Hypotheses
H_0: $\mu \geq \mu_0$



composite null hypothesis approach. Although this result might appear to indicate that the authors are split fairly equally regarding which approach to use, this conclusion would be wrong. As the data in Table 1 show, the textbooks that explain the statistical foundations underlying the composite null hypothesis approach generally do not use it, whereas the textbooks that use the composite null hypothesis approach generally do not explain its foundations! Of the 15 text-

TABLE 1
Use and Discussion of the Composite Null Hypothesis Approach

Discussion of the composite null hypotheses	Approach used: Simple	Approach used: Composite	Total
Discussion	11	4	15
No discussion	9	20	29
Total	20	24	44

Source: The authors' survey of 44 introductory business and economics statistics textbooks.

books that explicitly discuss the statistical foundations of the composite null hypothesis approach, only 4 of these actually use it; 11 of these choose, instead, to use the simple null hypothesis approach.[6] Alternatively, whereas 24 textbooks use the composite null hypothesis form, only 4 of these actually provide students with its statistical explanation. In contrast, 20 of these books discuss neither the multiplicity of sampling distributions nor the appropriate interpretation of the probability of a Type I error associated with the selected rejection region. Instead, they provide a statistical explanation suitable only for the simple null hypothesis approach.[7] Thus, if we were to characterize these textbooks by the statistical explanation used, rather than the form of the null hypothesis used, we would conclude that 40 of the 44 textbooks use the simple null hypothesis approach, and only 4 use the composite null hypothesis approach.

These results indicate important statistical and pedagogical problems with the composite null hypothesis approach as presented in many introductory business and economics statistics textbooks. The vast majority of textbooks that use the composite null hypothesis approach do not provide the appropriate statistical explanation that underlies this approach; they use, instead, an explanation suitable only for the simple null hypothesis approach.

SUMMARY

Authors of introductory economics and business statistics textbooks, and the instructors who use these textbooks, face a difficult task. They must explain and illustrate complicated mathematical and statistical concepts to students who typically find these concepts difficult to comprehend and to use. There is a significant difference of opinion among these authors over the best way to explain one-tailed hypothesis tests. Approximately half of them use the simple null hypothesis approach, and the rest use the composite null hypothesis approach.

Unfortunately, as our survey of these textbooks indicates, few textbook authors who use the composite null hypothesis approach provide students with the necessary statistical foundation to understand it. Instead, most of these textbooks contain the explanation suitable only for the simple null hypothesis approach. Fortunately, this problem can be solved fairly easily either by changing the form of the null hypothesis from composite to simple to match the statistical explanation or by changing the statistical explanation to that appropriate for the composite null hypothesis approach.

NOTES

1. For survey results on the most difficult statistics topics, see Aczel (1995,viii).
2. A list of the 44 books in our survey is available from the authors on request.
3. The differences between these two approaches hold even if the Z or t test were used. The student must still decide which value of μ to use in the test statistic.
4. The discussion of multiple sampling distributions under the null hypothesis and their associated Type I errors, as illustrated in Figure 2,is comparable to similar discussions of the multiple sampling distributions under the alternative hypothesis and their associated Type II errors that appear in most introductory textbooks.
5. In mathematical terms, the probability of a Type I error under the composite null hypothesis approach is a function of the unspecified values for the parameter of interest (see Kendall and Stuart 1973,196). Therefore, the rejection region, S, is chosen such that

   $$\max_{\mu \in H_0} P(\overline{x} \in S) = \alpha$$

   (Lehmann 1986, 69).
6. In their coverage of the composite null hypothesis approach, four textbooks discuss the multiple sampling distributions of the sample mean; five discuss the maximum probability of the Type I error; and six discuss issues related to these two.
7. For example, Johnson (1992) and Mann (1995) use notation similar to H_0: $\mu = 50$ ($\leq$), which,presumably, enables them to use the simple null hypothesis approach but discuss the results as if they had used the composite null hypothesis approach. Similarly, when Mason and Lind switched from the simple null hypothesis approach in their previous edition (1993) to the composite null hypothesis approach in their latest edition (1996), they merely added the appropriate inequality to the equal sign in the null hypothesis.

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