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Stock Market Volatility and Regime Shifts in Returns

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ABSTRACT

This paper relates variation in stock market volatility to regime shifts in stock market returns. We apply a Markov switching model to market returns and examine the variation in volatility in different return regimes. We find that stock returns are best characterized by a model containing six regimes with significantly different volatility across the regimes. Volatility is higher when returns are either above or below the normal regime—the further returns deviate from the normal regime, the higher the volatility. Furthermore, volatility is higher in negative return regimes than in positive return regimes. These observations lead us to conclude that return and volatility are related nonlinearly and that the relationship is asymmetric.

1. INTRODUCTION

The October 1987 stock market crash and a series of sharp breaks in stock prices that followed heightened concern about stock market volatility. Large changes in volatility can change investor's perceptions regarding risk, which will likely affect the desired rate of capital accumulation and

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rate of economic growth.[1] Despite the public concern, several studies by Schwert [21, 22] find that stock market volatility has not increased in recent years. However, since Mandelbrot's paper [16], numerous studies indicate that the volatility of stock returns varies over time. The attempt to explain this variation has led to fairly wide use of Engle's [7] autoregressive conditional heteroskedastic (ARCH) model and its derivatives. Recently, variation in volatility has been associated with regime shifts. For example, Glosten, Jagannathan, and Runkle [11, p. 1789] speculate on this possibility by suggesting the data may be explained by "...different regimes in which variance is relatively persistent but there are frequent and relatively unpredictable regime shifts."[2]

This paper relates variation in stock volatility to regime shifts in returns. In particular, we apply Hamilton's [12] Markov switching model to the time series of monthly stock market returns and examine the variation in volatility in different return regimes. We find that stock returns are best characterized by a model containing six regimes. Volatility is significantly higher when returns are either above or below "normal," which suggests a nonlinear relationship between returns and volatility. Furthermore, the relationship between returns and volatility is asymmetric. Volatility is higher in negative return regimes than in positive return regimes. These results may help explain the inverse relationship between returns and volatility that emerges in some estimates when a straight line is forced through the data.

The paper is organized as follows. Section 2 describes the two-stage estimation procedure we use to relate volatility with regime shifts in returns. Section 3 presents our empirical results and Section 4 concludes the paper.

2. TWO-STAGE ESTIMATION APPROACH

This paper uses a two-stage estimation procedure to relate volatility and returns. Markov switching models with two to seven regimes are applied to

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[1] Markowitz ([17], p. 89) notes that, "The concepts 'yield' and 'risk' appear frequently in financial writings. Usually if the term 'yield' were replaced by ... 'expected return,' and 'risk' by 'variance of return,' little change of apparent meaning would result." We use the term "volatility" to encompass all the different methods that are used to measure the variation in stock returns.

[2] See, as well, Hamilton and Susmel [13] and Turner, Startz, and Nelson [24].

stock returns in the first stage. The second stage estimates a volatility equation given different return regimes derived from the first stage.

Assume that all currently available information at time t is summarized in a state variable S_t, taking a value of $1, 2, \ldots,$ or k, i.e., $S_t \in \{1, 2, \ldots, k\}$. Since state variable S_t is unobservable, it is usually assumed that S_t evolves as a first-order Markov chain:

$$P(S_t = j | S_{t-1} = i) = p_{ij} \quad \text{for } i, j = 1, 2, \ldots k, \tag{1}$$

where the p_{ij} form the transition probability matrix P, $P = [p_{ij}]$. These are the probabilities associated with moving to, say, state 2 (or 1 or 3, etc.) next period given that state 1 (or 2 or 3, etc.) currently prevails.

Let R_t be the rate of return generated by the market portfolio from time $t-1$ to time t. The expected return in terms of Hamilton's [12] Markov switching model can be written as

$$E(R_t) = X_t'\beta + \phi_1(R_{t-1} - X_{t-1}'\beta) + \cdots + \phi_q(R_{t-q} - X_{t-q}'\beta), \tag{2}$$

where X_t is a vector which contains state dummy variables $I_{i,t}$, $i = 1, 2, \ldots, k$, and β contains the mean return in each state. The state dummy variable $I_{i,t}$ takes a value of 1 when $S_t = i$ and 0 otherwise.

The first stage of our estimation procedure uses Hamilton's [12] method to estimate the parameters (ϕ, β, P) in (1) and (2). We consider various models with two to seven regimes and one to four lags. The Akaike Information Criterion (AIC) as in Sclove [23] is used to determine the optimum number of regimes and lags.[3]

In the second stage, the volatility equation is considered. We follow Officer [19], Black [2], Merton [18], Christie [5], Schwert [21], and Whitelaw [25] in using the sample data to measure volatility. In our case, monthly volatility is measured by the standard deviation of daily returns. To examine the relationship between volatility and the different return regimes, we consider the following volatility equation:

$$V_t^r = D_t'\gamma + Z_t'\delta + \eta_t, \tag{3}$$

[3]Hansen [14] suggests an alternative test of the model. However, this type of test involves the difficult issue of unidentified parameters under the alternative and it is far more complex than the AIC model selection procedure.

where D_t contains the regime dummy variables, Z_t contains lagged volatility, and η_t is a random error. The regime dummy variables are generated by classifying the return in each month by regime. Suppose the optimum number of regimes and lags in the regime shift model for returns is determined. The probabilistic inferences about the regimes that the various months belong in are:

$$P(\hat{S}_t) \equiv p(S_t / R_{t+q}, R_{t+q-1}, \ldots, R_{-q}, \hat{\theta}_T), \tag{4}$$

where q is the number of lags used in (2) and $\hat{\theta}_T$ is the set of coefficients from the first-stage estimation, including the estimates of β, ϕ, and P. Equation (4) gives the probabilities that a given month belongs in the various regimes. This permits us to assign observed monthly returns to various regimes. Specifically, month t is assigned to regime i if regime i has the highest smoothed probability among the k regimes. Using this regime classification, the regime dummy variables are defined as follows:

$$D_{i,t} = 1 \quad \text{if } P(\hat{S}_t = i) > P(\hat{S}_t = j), \qquad \text{for all } j \neq i \quad \text{and} \tag{5}$$

$$D_{i,t} = 0 \quad \text{if } P(\hat{S}_t = i) \leqslant P(\hat{S}_t = j), \qquad \text{for any } j \neq i. \tag{6}$$

Then D_t in (3) is defined as $D_t = [D_{1,t}, D_{2,t}, \ldots, D_{k,t}]$. The parameters of the volatility equation are estimated by ordinary least squares. The estimates of γ show the levels of volatility in different return regimes.

3. EMPIRICAL RESULTS

We apply our model to the value-weighted New York Stock Exchange index from July 1962 to December 1993. The data are obtained from the CRSP data tape. Monthly returns are used for the first-stage estimation. Schwert's [21] measure of monthly volatility is used for the second-stage estimation. Denote R_{it} as the daily return in month t. Then monthly volatility, V_t^r, is

$$V_t^r = \sqrt{\sum_{i=1}^{N_t} (R_{it} - \overline{R_t})^2},$$

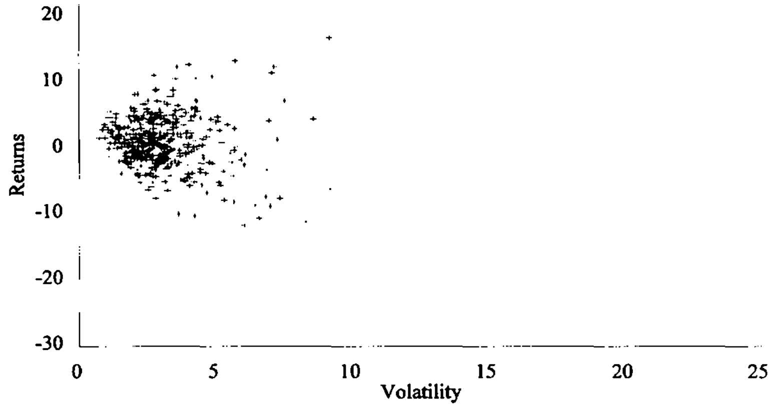

Fig. 1. Stock returns and volatility 1962.07–1993.12.

where N_t is the number of daily returns and $\overline{R_t}$ is the average return in month t. Figure 1 shows the plot of monthly stock returns against volatility.

3.1. ESTIMATING THE EXPECTED RETURN EQUATION AND REGIME CLASSIFICATION

In the first stage, we apply the maximum likelihood procedure and Hamilton's [12] filtering algorithm to estimate (1) and (2). Table 1 presents the parameter estimates for the return equation with four lags and two to seven regimes. The numbers in the parentheses under the lag coefficients, ϕ, are standard errors. The numbers in square brackets below the β coefficients indicate the number of months that fall into that regime. For example, in the case of a two-regime model, the mean return in regime 1 is 1.29% per month and 361 months fall into this regime, while the mean return is -9.11% per month in regime 2 with 9 months falling into this regime.

Previous applications of the Markov switching model to the return equation restrict the analysis to two regimes. While these studies use different sample periods and slightly different financial series than used here, our Table 1 result for the two-regime case (see regression 2) is similar to those obtained previously. Like them, we find one regime in which the mean return is positive and one in which it is negative and many

TABLE 1

Estimates of the Return Equation for Four-Lag Markov Switching Models

Coefficient	Regimes 2	3	4	5	6	7
ϕ_1	−0.0330	−0.0060	−0.1598	−0.2995	−0.3760	−0.4462
	(0.0423)	(0.0480)	(0.0484)	(0.0596)	(0.0605)	(0.0658)
ϕ_2	−0.0286	−0.0418	−0.1663	−0.2325	−0.2688	−0.2874
	(0.0396)	(0.0455)	(0.0507)	(0.0763)	(0.0735)	(0.0642)
ϕ_3	0.0218	0.0433	−0.0719	−0.0337	0.0239	0.0276
	(0.0426)	(0.0558)	(0.0550)	(0.0742)	(0.0642)	(0.0703)
ϕ_4	0.0318	−0.0195	−0.0949	−0.1374	−0.0973	−0.1578
	(0.0397)	(0.0495)	(0.0525)	(0.0609)	(0.0599)	(0.0606)
β_1	1.2886	8.3212	10.7693	11.6572	11.1957	14.8234
	[361]	[17]	[15]	[13]	[15]	[1]
β_2	−9.1114	0.8980	1.7783	3.9251	4.1355	11.0588
	[9]	[339]	[227]	[120]	[117]	[12]
β_3		−8.4690	−0.6107	1.0549	1.0551	4.2539
		[14]	[116]	[100]	[99]	[118]
β_4			−8.8932	−1.8873	−1.7193	0.9541
			[12]	[124]	[118]	[106]
β_5				−9.4258	−7.3269	−2.0203
				[13]	[20]	[118]
β_6					−21.2665	−7.9122
					[1]	[14]
β_7						−20.8122
						[1]

Note: The expected return equation is $E(R_t) = X_t'\beta + \phi_1(R_{t-1} - X_{t-1}'\beta) + \cdots + \phi_4(R_{t-4} - X_{t-4}'\beta)$, where X_t contains state dummy variables such that $X_t' \equiv [I_{1,t}, I_{2,t}, \ldots, I_{k,t}]$. The numbers in the parentheses under ϕ coefficients are standard errors. The numbers in the brackets under β coefficients are the numbers of months assigned to each regime.

times lower than the return in the positive regime.[4] However, this two-regime restriction does not conform well with the data because it contains the implicit assumption that all extreme outcomes for monthly stock returns have the same sign. This is clearly rejected by the data when the two-regime restriction is relaxed. Table 2 shows the AIC values for two to seven regimes and one to four lags. They suggest that relaxing the two-regime restriction results in identifying six regimes and four lags for this data set. In this six-regime model, 15 months fall into regime 1 (extreme positive

[4] See, for example, Turner Startz, and Nelson [24], Cecchetti, Lam, and Mark [4], and Hung [15].

TABLE 2
AIC Values for Markov Switching Models

Regimes	1 Lag	2 Lags	3 Lags	4 Lags
2	3633	3628	3617	3612
3	3613	3608	3596	3592
4	3604	3591	3582	3575
5	3569	3552	3543	3536
6	3527	3501	3492	3488*
.7	3535	3518	3508	3499

Asterisk indicates the lowest AIC value.

returns of 11.20% per month), 20 months fall into regime 5 (extreme negative returns of −7.33% per month), and one month (October 1987) falls into regime 6 with a return of −21.27%. The number of months falling into more moderate return regimes is much larger (117, 99, and 118 months). Furthermore, we are interested in analyzing the relationship between expected return and volatility. Confining the analysis to only two regimes restricts the relationship between expected return and volatility to one that is linear. This is overly binding given the results that are obtained when this assumption is relaxed.

Table 3 reports estimates of the transition probabilities for the six-regime model. The data indicate that the "normal" regime (regime 3 with a mean monthly return of 1.06%) is very stable. For example, if the current month falls in the normal regime, the probability is more than 92% that the following month will also fall in the normal regime. However, if the current month falls outside the normal regime, the probability of returning to the normal regime next month is typically very small, except in the case of regime 1.

TABLE 3
Estimated Markov Probabilities for the Six-Regime and Four-Lag Model

Regime at time t	Regime at time $t+1$					
	1	2	3	4	5	6
1	0.0000	0.2508	0.5523	0.1968	0.0000	0.0001
2	0.0000	0.5448	0.0035	0.4411	0.0106	0.0001
3	0.0097	0.0055	0.9241	0.0607	0.0000	0.0000
4	0.1054	0.2970	0.0000	0.4758	0.1139	0.0079
5	0.0725	0.5448	0.0000	0.1619	0.2207	0.0001
6	0.0003	0.0000	0.0001	0.0014	0.9978	0.0004

3.2. *ESTIMATING THE VOLATILITY EQUATION*

In the second stage, we estimate the parameters of the volatility equation. Table 4 shows the least-squares estimation results for (3) and contrasts them with several other specifications. The results shown in the first column of Table 4 are obtained for a simple autoregression of order 6.

TABLE 4
Estimates of the Volatility Regression

	Regression				
Regressors	1	2	3	4	5
Constant	0.9143	0.9085	0.9775	0.9706	0.7933
	(0.2211)	(0.2078)	(0.1582)	(0.1544)	(0.1425)
V_{t-1}^r	0.3344	0.2988	0.3087	0.2932	0.1980
	(0.0523)	(0.0494)	(0.0374)	(0.0367)	(0.0343)
V_{t-2}^r	0.1965	0.2353	0.1824	0.2021	0.1933
	(0.0550)	(0.0520)	(0.0394)	(0.0387)	(0.0346)
V_{t-3}^r	0.1028	0.1266	0.1462	0.1549	0.1640
	(0.0554)	(0.0522)	(0.0397)	(0.0388)	(0.0348)
V_{t-4}^r	−0.1075	−0.0815	−0.0711	−0.0610	−0.0712
	(0.0553)	(0.0522)	(0.0397)	(0.0388)	(0.0348)
V_{t-5}^r	0.0761	0.0549	0.0576	0.0486	0.0386
	(0.0547)	(0.0515)	(0.0392)	(0.0383)	(0.0342)
V_{t-6}^r	0.1155	0.1207	0.0579	0.0641	0.0577
	(0.0520)	(0.0489)	(0.0374)	(0.0365)	(0.0327)
R_t		−0.1196		−0.0577	
		(0.0172)		(0.0133)	
$D_{1,t}$					1.5878
					(0.2728)
$D_{2,t}$					0.3678
					(0.1395)
$D_{4,t}$					0.6486
					(0.1320)
$D_{5,t}$					2.3604
					(0.2418)
$D_{6,t}$			19.9808	18.6883	20.6641
			(1.0756)	(1.0909)	(0.9420)
$\bar{R}^2$	0.2941	0.3761	0.6386	0.6557	0.7259
$\chi^2(24)$	12.54	16.54	26.00	21.28	19.18
	(0.97)	(0.87)	(0.35)	(0.62)	(0.74)

Note: R_t is the monthly return. $D_{6,t}$ is also the dummy variable for October 1987. $\chi^2(24)$ is the Ljung-Box statistic with 24 lags of residuals. The numbers in the parentheses under the estimated coefficients are the standard errors. The numbers in the parentheses under χ^2 are the p-values.

For purposes of comparison, the specifications in columns 2–4 add the monthly return and/or a dummy for October 1987. The last column shows the results obtained when the six regime dummy variables, $D_{j,t}$, $j = 1, 2, \ldots, 6$, generated in stage 1 are included in the regression. The regime dummy variable for the normal regime, $D_{3,t}$, is suppressed in these estimates. Its coefficient is measured by the estimate of the constant term. The Ljung-Box test statistics $\chi^2(24)$ for the residuals are insignificant for each of the Table 4 regressions, which implies that the appropriate number of autoregressive lags are included in each.

3.3. *INTERPRETING THE RESULTS*

There are several important points that can be drawn from the Table 4 comparisons. Our first-stage estimation indicates that October 1987 is an outlier since regime 6 contains only that observation. It is an important outlier because the results in Table 4 suggest that this one observation accounts for a considerable amount of the variation in volatility. For example, the $\bar{R}^2$ in the regressions that control for this outlier (regressions 3, 4, and 5) are about twice as high as the others. These results show that including an October dummy in the regression will understate the unusual behavior in October 1987 and overstate the effect of other Octobers.[5] Second, the equation (3) result shown by regression 5 fits the data best since its $\bar{R}^2$ is the highest. The coefficients of the regime dummy variables are all significantly positive in this regression, which means that volatility is higher whenever market returns deviate from the normal regime in either direction. Finally, regressions 2 and 4 indicate that returns and volatility are negatively related in linear specifications, which is what some others have found. However, regression 5 implies that returns and volatility are related nonlinearly. This can be seen by comparing the estimated coefficients of β and γ for expected returns and volatility across the six regimes.

Table 5 summarizes estimates of β and γ from Tables 1 and 4 for the six-regime model. It shows that volatility in regime 1 (the regime in which mean return is the highest) is higher than in regime 2 and is higher in regime 2 than in regime 3 (the normal return regime). Returns and volatility are positively related across regimes 1, 2, and 3. On the other hand, volatility increases in regimes 4–6 as mean returns fall below normal. It is highest in regime 6 where the mean return is lowest. Returns and volatility are negatively related across regimes 3, 4, 5, and 6. As shown

[5] See, for example, Glosten, Jagannathan, and Runkle [11].

TABLE 5
The Expected Returns and Volatility in Different Regimes for Six-Regime and Four-Lag Markov Switching Model

Regime i	Expected returns (β_i)	Expected volatility (γ_i)	Number of months
1	11.1957	2.3811	15
2	4.1355	1.1611	117
3	1.0551	0.7933	99
4	−1.7193	1.4419	118
5	−7.3269	3.1537	20
6	−21.2665	21.4573	1

Note: The expected returns are estimates of β for the model with six regimes in Table 1. The expected volatility are estimates of γ for the regression 5 in Table 4, where estimate of γ_3 is the constant and other estimates are adjusted for this constant term. The numbers in the last column are the numbers of months assigned to each regime.

in Figure 2, these results indicate that returns and volatility are nonlinearly related across the six regimes.[6] Furthermore, the relationship is asymmetric. This is seen by comparing the estimated coefficients of $D_{2,t}$ and $D_{4,t}$ as well as those of $D_{1,t}$ and $D_{5,t}$ shown in Table 4. Tests for the equality between these coefficients yield F statistics of 5.62 and 5.53. Both are significant at 5% level, indicating that expected volatility is significantly higher when returns are below normal compared to when they are above normal.

The above helps explain why forcing a straight line through the observations of returns and volatility produces the negative slope coefficient that has puzzled others. In addition, if stock returns are best characterized by a regime shift model, ignoring the different return regimes may have contributed to the observation of Mandelbrot [16], Fama [8], and Fama and Roll [9, 10] that the distribution of stock returns is fat-tailed. Furthermore, the asymmetry we note is likely related to skewness in the return distribution noted by Arditti [1], Blume [3], and Duffee [6].

4. CONCLUSION

This paper relates variation in the volatility of stock returns to regime shifts in returns. In particular, we apply a Markov switching model to stock

[6] Using different methodology, Pettengill, Sundaram, and Mathur [20] find a similar result for the relationship between systematic risk and return in the context of a CAPM model.

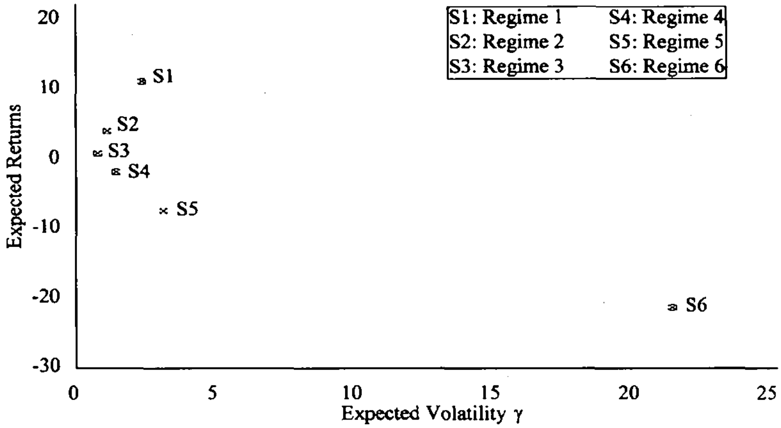

Fig. 2. Expected stock returns and volatility.

returns and examine the variation in volatility in different return regimes. We conclude that returns are best characterized by six regimes and that October 1987 is an outlier accounting for a large portion of the variation in volatility. In addition, our estimates are consistent with a strong contemporaneous relationship between returns and volatility across regimes with higher volatility when returns are either above or below "normal." Furthermore, the increase in volatility is larger for negative deviations in returns than for positive deviations. These observations lead us to conclude that return and volatility are related nonlinearly and that the relationship is asymmetric. If so, it helps explain the weak inverse relationship that others have found when a straight line is forced through the data.

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