Do it or Don't

Dr. Frank Harary
New Mexico State University

Mathematical games of achievement and avoidance have been formulated with play on graphs, groups, geometries, numbers, chess pieces, and theorems! These will be described and played. Unsolved problems are abundant.

Using Hamiltonian cycles to obtain an upper bound on the rope-length of knots

Claus Ernst
Western Kentucky University

The rope-length of a knot K is the minimal length of a rope (with unit thickness) one needs to tie the knot K. A knot projection can be thought of as a 4-regular planar graph. Hamiltonian cycles in such graphs can be used to obtain an upper bound on the rope-length of the knot.

On Regular Cayley Maps with Alternating Power Functions

John Martino, Western Michigan University
Paula Smith, Ohio Dominican University

Two classes of regular Cayley maps, balanced and antibalanced, have long been understood. A recent generalization is that of an e-balanced map. These maps can be described using the power function; e-balanced maps are the ones with constant power functions on the generating set. In this paper we examine a further generalization to the situation where the power function alternates between two values.

On γ-Labelings of Graphs

Gary Chartrand, Western Michigan University
David Erwin, Trinity College
Donald W. VanderJagt, Grand Valley State University
Ping Zhang*, Western Michigan University

Let G be a graph of order n and size m. A γ-labeling of G is a one-to-one function $f : V(G) \to \{0, 1, 2, \ldots, m\}$ that induces a labeling $f' : E(G) \to \{1, 2, \ldots, m\}$ of the edges of G defined by $f'(e) = |f(u) - f(v)|$ for each edge $e = uv$ of G. The value of a γ-labeling f is $(f) = \sum_{e \in E(G)} f'(e)$. The maximum value of a γ-labeling of G is defined as

$$_{\max}(G) = \max\{(f) : f \text{ is a } \gamma\text{-labeling of } G\},$$

while the minimum value of a γ-labeling of G is

$$_{\min}(G) = \min\{(f) : f \text{ is a } \gamma\text{-labeling of } G\}.$$

We present some results in this area.

Representations of disjoint unions of complete graphs:

Anthony B. Evans
Wright State University

A representation of a graph G modulo n is an assignment of distinct labels between 0 and $n-1$ to the vertices of G so that the difference of two labels is relatively prime to n if and only if the corresponding vertices are adjacent. The representation number of G is the smallest positive integer n for which G is representable modulo n. In this talk we will present some new bounds on representation numbers and product dimensions of disjoint unions of complete graphs. These representations are closely related to mutually orthogonal sets of latin squares.

On the existence of a rainbow 1-factor in 1-factorizations of K_{rn}^r

Saad I. El-Zanati*, Michael J. Plantholt Papa Amar Sissokho, Lawrence E. Spence,
Illinois State University

Let $\mathcal{F}$ be a 1-factorization of the complete uniform hypergraph $\mathcal{G} = K_{rn}^r$ with $r \geq 2$ and $n \geq 3$. We show that there exists a 1-factor of $\mathcal{G}$ whose edges belong to n different

1-factors in $\mathcal{F}$. Such a 1-factor is called a "rainbow" 1-factor or an "orthogonal" 1-factor. This answers a question attributed to Alex Rosa in 1977. In this talk we present the proof for complete graphs (the case $r = 2$) and provide an outline of the proof for the general hypergraph case.

Distance in Graphs - Taking the long view

Gary Chartrand
Western Michigan University

Several concepts and results dealing with distance in graphs will be presented.

Indecomposable Hypergraphs

Andrew C. Breiner
University of Nebraska

Let G be a finite graph, a subset X of V is an interval of G if for $a, b \in X$ and $x \in V \setminus X$, we have $(a, x) \in E$ if and only if $(b, x) \in E$. So $\emptyset$, V and $x \in V$ are trivial intervals of G. The graph G is said to be *indecomposable* if every interval is trivial. Let $X \subseteq V$, $H_k = \{V \setminus X, E_k\}$, is called a hypergraph of G, if the induced subgraph $G_{[X \cup F]}$ is indecomposable, $F \subsetneq V \setminus X$, $|F| = k$, and $F \in E_k$. In this paper, we want to characterize indecomposability of graphs in terms of their hypergraphs.

Some Results on Mouths and Ears in Polygon Visibility Graphs

Jay S. Bagga John W. Emert J. Michael McGrew* Frank W. Owens
Ball State University

A mouth in a simple polygon P is a vertex xi such that the interior of the triangle [xi-1, xi, xi+1] contains no vertices of P and the interior of the diagonal [xi-1, xi+1] lies entirely in the exterior of P. An ear in a simple polygon P is a vertex xi such that the interior of the diagonal [xi-1, xi+1] lies entirely in the interior of P. We discuss some properties of the ears and mouths of polygon visibility graphs, and conjecture that every simple polygon visibility graph with exactly 2 ears and 1 mouth (called an anthropomorphic polygon visibility graph) is a planar graph.

Some Results on the Crossing Numbers of Principal Polygon Visibility Graphs

Jay K. Bagga John W. Emert J. Michael McGrew Frank W. Owens*
Ball State University

A simple polygon P is called a principal polygon if each vertex of the polygon is a principal vertex, i.e., if each vertex of the polygon is either an ear or a mouth. A mouth chain in P is a maximal sequence of consecutive mouths. We obtain an upper bound on the crossing number of the polygon visibility graph in terms of the number of mouth chains and their lengths. These upper bounds are sharp for any number of mouth chains of arbitrary lengths.

Two-Path Convexity in Multipartite Tournaments

Darren Parker
University of Dayton

Convexity in directed and undirected graphs is widely studied. In 1970, J. Pfaltz introduced path convexity in directed graphs. Soon after, Erdös, Fried, Hajnal, and Milner, along with Moon studied two-path convexity in tournaments. Harary and Nieminen introduced geodesic convexity in undirected graphs in 1981. This led to many forms of convexity in graphs, including induced path convexity and triangle path convexity. This talk focuses on two-path convexity in multipartite tournaments. In particular, we study a parameter called breadth. While breadth has its origins in lattice theory, it is strongly related to a parameter called the hull number, which Everett and Seidman defined while studying geodesic convexity.

An algorithm randomly generates 4-regular planar maps which contain a Hamiltonian cycle

Uta Ziegler
Western Kentucky University

An algorithm is presented which randomly generates 4-regular planar maps which contain a Hamiltonian cycle. Maps with a Hamiltonian cycle can be used to determine an upper bound for the rope length of thick knots. The worst case run-time for the algorithm is O(N*N*N), although empirical data shows an average runtime of O(N*N).