# MIGHTY (MIdwest GrapH TheorY) Conference XXXIV

## Saturday, 13 October 2001

## Department of Mathematics and Statistics

## Oakland University

## Rochester, Michigan

**Organizing Committee**:


- Eddie Cheng (Co-chair) *echeng@oakland.edu*
- Curt Chipman *chipman@oakland.edu*
- Jerry Grossman *grossman@oakland.edu*
- Marc Lipman (Co-chair) *lipman@oakland.edu*
- Lazaros Kikas *lkikas@oakland.edu*
- Serge Kruk *kruk@oakland.edu*

## Schedule

**All talks are 20 minutes long plus 5 minutes for questions.**

- 8:00–8:40: Registration
- 8:40–9:00: Welcome and refreshments
- 9:00–9:25: Maximal Independent Sets in Graphs with at Most $r$ Cycles, *Bruce Sagan (Michigan State University)*
- 9:25–9:50: Meets and Joins in the Poset of Graphs of Order $n$, *Roger Eggleton (Illinois State University)*
- 9:50–10:15: Spreading Multiple Team Meetings in a Sports Schedule, *Mike Plantholt (Illinois State University)*
- 10:15–10:30: Break
- 10:30–10:55: Chordally Signed Graphs, *Terry McKee (Wright State University)*
- 10:55–11:20: Matroids Coming from Graphs and Linear Transformations, *Dan Slilaty (Wright State University)*
- 11:20–11:45: Multidesigns of the Complete Graph, *Atif Abueida and Mike Daven (University of Dayton)*
- 11:45–1:30: Lunch
- 1:30–1:55: Nowhere Zero 4-flows in Regular Matroids, *Hong-Jian Lai (West Virginia University)*
- 1:55–2:20: Supereulerian Planar Graphs, *Mingquan Zhan (West Virginia University)*
- 2:20–2:45: An $s$-Hamiltonian Line Graph Problem, *ZhiHong Chen (Butler University)*
- 2:45–3:10: Graph Homomorphism into an Odd Cycle, *Gexin Yu (West Virginia University)*

- 3:10–3:30: Break
- 3:30–3:55: Terrain Model Acquisition by Robot Teams, *Nachimuthu Manickam (DePauw University)*
- 3:55–4:20: Bipartite Rainbow Ramsey Numbers, *Linda Eroh (University of Wisconsin–Oshkosh)*
- 4:20–4:45: Triangle-Free Regular Graphs as an Extremal Family, *Kenneth J. Roblee (Youngstown State University)*
- 4:45–5:10: Some New Results in Segment Endpoint Visibility Graphs, *Jay Bagga (Ball State University)*
- 5:10–5:35: A Disjoint Path Problem for the Alternating Group Graph, *Lazaros Kikas (Oakland University)*

9:00–9:25

**Maximal Independent Sets in Graphs with at Most $r$ Cycles**, *Bruce Sagan (Michigan State University)*

We find the maximum number of maximal independent sets in two families of graphs: all graphs with $n$ vertices and at most r cycles, and all such graphs that are also connected. In addition, we characterize the extremal graphs. This proves a strenghtening of a conjecture of Goh and Koh. This is joint work with Vincent Vatter.

9:25–9:50

**Meets and Joins in the Poset of Graphs of Order $n$**, *Roger Eggleton (Illinois State University)*

If $G$ is a spanning subgraph of a simple graph $H$, we write $G \leq H$ and call $G$ a *reduction* of $H$, and $H$ an extension of $G$. Under this binary relation, the set $\mathcal{G}(n)$ of all simple graphs of order $n$ is a poset. A subset $\mathcal{J} \subseteq \mathcal{G}(n)$ is an ideal if it is closed under "betweenness" that is, if $G$ and $H$ are in $\mathcal{J}$, then every $G'$ satisfying $G \leq G' \leq H$ is also in $\mathcal{J}$. It turns out that $\mathcal{G}(n)$ is a lattice just when $n \leq 4$. Nevertheless, in terms of the ideals in $\mathcal{G}(n)$ there are natural notions of *meet* and *join* for any set $S$ of simple graphs of order $n$. In particular, the join $\vee S$ turns out to be the set of *minimal spanning-universal graphs* for $S$, that is, the minimal graphs which contain each graph in S as a spanning subgraph. The join $\vee S$ is always an *independent* set in $\mathcal{G}(n)$, that is, none of its elements is an extension of any other. I will discuss these notions, and present several concrete results, particularly for cases where $S$ is the set of all trees of order $n$, or a subset of the trees. A convenient context for studying joins (and meets) in $\mathcal{G}(n)$ is provided by the *join digraph* $\mathcal{J}\mathcal{G}(n)$. Its vertices are all independent subsets of graphs of order $n$, and $S \longrightarrow \mathcal{J}$ is a directed edge precisely when $\mathcal{J} = \vee S$ I will present some recent results on the structure of this digraph. This is joint work with Peter Adams, James MacDougall, and Ebadollah Mahmoodian.

9:50–10:15

**Spreading Multiple Team Meetings in a Sports Schedule**, *Mike Plantholt (Illinois State University)*

In most professional sports leagues, teams meet each other multiple times during a season, but that number varies among pairs of teams. How can the schedule be arranged to spread those meetings somewhat homogeneously over the season? We review some related theorems, and give a new result in this area.

10:30–10:55

**Chordally Signed Graphs**, *Terry McKee (Wright State University)*

A *chordally signed graph* is a signed chordal graph (meaning that each edge is designated as being positive or negative and every induced cycle is a triangle) in which every positive cycle $C$ (meaning every cycle $C$ that contains an even number of negative edges) has a chord $e$ such that $C \cup \{e\}$ forms two positive cycles. Two characterizations of chordally signed graphs support the naturalness of this definition. In addition, chordally signed graphs can be easily recognized when the underlying graph has at most two maximal complete subgraphs (with minimal balancing vertex sets playing a key role).

10:55–11:20

**Matroids Coming from Graphs and Linear Transformations**, *Dan Slilaty (Wright State University)*

There are many different ways of defining a matroid on the edge set of a graph $G$. One of the simplest matroids on the edge set $E(G)$ is defined by setting the independent subsets of $E(G)$ to be the edge sets of forests in $G$. Equivalently one may define the same matroid by setting the circuits in $E(G)$ to be the edge sets of simple closed paths in $G$. This matroid is often called the cycle matroid or polygon matroid of $G$.

We will discuss a method of defining matroids on $E(G)$ using linear transformations of the cycle space of $G$ into vector spaces over $GF(2)$. These matroids turn out to be what matroid theorists would call a lift of the cycle matroid. These matroids also have very desirable topological properties which, time permitting, we will discuss.

11:20–11:45

**Multidesigns of the Complete Graph**, *Atif Abueida and Mike Daven (University of Dayton)*

A graph decomposition is a partition of the edges of the complete graph into some subgraph $G$. Let $G$ and $H$ be a pair of non-isomorphic graphs on fewer than $m$ vertices. We introduce several new problems about decomposing the complete graph $K_m$ into copies of $G$ and $H$. We also begin to examine variations to the problems of subgraph packing, covering, and factorization.

1:30–1:55

**Nowhere Zero 4-flows in Regular Matroids**, *Hong-Jian Lai (West Virginia University)*

Jensen and Toft in 1995 conjectured that every 2-edge-connected graph without a $K_5$-minor has a nowhere zero 4-flow, which is equivalent to saying that every 2-edge-connected graph without a $K_5$-minor has a 3-cycle 2-cover. Walton and Welsh in 1980 proved that if coloopless binary matroid $M$ does not have a minor in $\{F_7^*, M(K_{3,3}), M^*(K_5)\}$, then $M$ admits a nowhere zero 4-flow. In this note, we prove that if a binary coloopless matroid $M$ does not have a minor in $\{F_7^*, M(K_5), M^*(K_5)\}$, then $M$ has a 3-cycle 2-cover. Our result implies the Jensen and Toft conjecture. This is joint work with Haifung Poon.

1:55–2:20

**Supereulerian Planar Graphs**, *Mingquan Zhan (West Virginia University)*

We investigate supereulerian graph problems within planar graphs. We prove that if a 2-edge-connected planar graph $G$ is at most three edges short of having two edge-disjoint spanning trees, then $G$ is supereulerian except for a few classes of graphs. This is applied to show the existence of spanning Eulerian subgraphs in planar graphs with small edge cut conditions. We also determine several extremal bounds for planar graphs to be supereulerian. This is joint work with Hong-Jian Lai (West Virginia University).

2:20–2:45

**An $s$-Hamiltonian Line Graph Problem**, *ZhiHong Chen (Butler University)*

For an integer $k > 0$, a graph $G$ is $k$-triangular if every edge of $G$ lies in at least $k$ distinct 3-cycles of $G$. In [*J. Graph Theory*, **11** (1987), 399–407], Broersma and Veldman proposed an open problem: For a given positive integer $k$, determine the value $s$ for which the statement "the line graph $L(G)$ of a $k$-triangular graph $G$ is $s$-hamiltonian if and only $L(G)$ is $(s+2)$-connected" is valid. Broersma and Veldman proved in 1987 that the statement above holds for $0 \le s \le k$ and asked, specifically, whether the statement holds when $s = 2k$. In this paper we prove that the statement above holds for $0 \le s \le \max\{2k, 6k-16\}$. This is joint work with Hong-Jian Lai (West Virginia University), Peter C. B. Lam (Hong Kong Baptist University), and Deying Li (Central China Normal University).

2:45–3:10

**Graph Homomorphism into an Odd Cycle**, *Gexin Yu (West Virginia University)*

For graphs $G$ and $H$, a map $f : V(G) \mapsto V(H)$ is a homomorphism if $f$ preserves adjacency. Let $Hom(G, H)$ denote the set of all homomorphisms from $G$ into $H$. In this paper we prove that for a graph $G$ with $n = |V(G)|$ and for $k$ with $n \geq k \geq 5$, if the odd girth of $G$ is at least $2k + 1$ and if the minimum degree $\delta(G) > 2n/(2k + 3)$, then $Hom(G, Z_{2k+1}) \neq \emptyset$, where $Z_{2k+1}$ denotes the cycle of length $2k + 1$. As a corollary, we settled affirmatively the following open problem posted by Alberson, Chan, and Haas in 1993: If a graph $G$ satisfies the conditions above, must the independence of $G$, which is the ratio of the independence number of $G$ to the number of vertices of $G$, be at least $k/(2k + 1)$? This is joint work with Hong-Jian Lai (West Virginia University).

3:30-3:55

**Terrain Model Acquisition by Robot Teams**, *Nachimuthu Manickam (DePauw University)*

We address the model acquisition problem for an unknown planar terrain by a team of robots. The terrain is cluttered by a finite number of polygonal objects whose shapes and positions are unknown. The robots are point sized and equipped with visual sensors which acquire all visible parts of the terrain by scan operations executed from their locations. The robots communicate with each other via wireless connection. The performance is measured by the number of sensor operations, which are assumed to be the most time-consuming of all robot operations. We employ visibility graph methods in hierarchical setup. For terrains with convex obstacles the sensing time can be shown to be 1/n of that of a single robot case for $n = 2, 3$ and 4.

3:55–4:20

**Bipartite Rainbow Ramsey Numbers**, *Linda Eroh (University of Wisconsin–Oshkosh)*

We say that an edge-colored graph is *rainbow* if every edge is a different color and *monochromatic* if every edge is the same color. For bipartite graphs $G_1$ and $G_2$, the *bipartite rainbow ramsey number* $BRR(G_1, G_2)$ is the smallest integer $N$ such that any edge-coloring of the complete bipartite graph $K_{N,N}$ with any number of colors must contain either a monochromatic subgraph isomorphic to $G_1$ or a rainbow subgraph isomorphic to $G_2$. This number exists if and only if $G_1$ is a star or $G_2$ is a star forest. We consider some bounds and formulas for this number when one of $G_1$ and $G_2$ is a star and the other is a star, matching, path, or 4-cycle. This is joint work with Ortrud Oellermann (University of Winnipeg).

4:20–4:45

**Triangle-Free Regular Graphs as an Extremal Family**, *Kenneth J. Roblee (Youngstown State University)*

It has been shown that if $G = (V, E)$ is a simple graph with $n$ vertices, $m$ edges, an average (per edge) of $t$ triangles occurring on the edges, and $J = \max_{uv \in E}|N(u) \cup N(v)|$, then $4m \leq n(J + t)$.

For $J = n - 2$ and $t = 0$, it has recently been shown that the only extremal graph (except when $n = 8, 10$) for this inequality is $K_{\frac{n}{2},\frac{n}{2}} - $(1-factor). Here, we use a well-known theorem of Andrásfai, Erdős, and Sós to characterize the extremal graphs for $t = 0$, any given value of $n - J$, and $n$ sufficiently large (they are the regular bipartite graphs). Then we give some examples of extremal non-bipartite graphs for smaller values of $n$.

4:45-5:10

**Some New Results in Segment Endpoint Visibility Graphs**, *Jay Bagga (Ball State University)*

We review some properties of segment endpoint visibility graphs and present some new results. We extend this concept to a class of segment endpoint visibility graphs on the sphere. We also describe some current work in progress for these classes of graphs. This is joint work with John Emert (Ball State University), and J. Michael McGrew (Ball State University).

5:10-5:35

**A Disjoint Path Problem for the Alternating Group Graph**, *Lazaros Kikas (Oakland University)*

A graph has the $k$-disjoint path property if for every $k$ disjoint vertex pairs $\{s_1, t_1\}, \ldots, \{s_k, t_k\}$ there exist $k$ vertex-disjoint paths, one connecting each pair. It has been shown that the $k$-disjoint path property is NP-complete when $k \geq 3$. A necessary condition for any graph to satisfy the $k$-disjoint path property is that it is $(2k-1)$-connected. If a $(2k-1)$-connected graph has the $k$-disjoint path property, then it has the *maximum disjoint path property*. Gu and Peng (1998) showed that the star graphs have the maximum disjoint path property and gave an efficient algorithm for finding these disjoint paths. Cheng and Lipman (1999) proved that split star graphs also possess the maximum disjoint path property. I will discuss my current work to use their strategy to prove that the alternating group graph also possesses the maximum disjoint path property. This is joint work with Eddie Cheng (Oakland University).