# Fiftieth MIdwest GrapH TheorY Conference

University of Wisconsin – Superior October 22-23, 2010

# MIGHTY L

Program

Abstracts

**Participants** 

Final Version

Organizing Committee: Sergei Bezrukov (UWS) Dalibor Fronček (UMD) Uwe Leck (UWS) Steve Rosenberg (UWS)

# Program

#### Friday, October 22

7:00 – 9:00 pm: Reception and Buffet Yellowjacket Union (YU) Landing

### Saturday, October 23

All talks will take place in the YU, Room 203.

9:00 - 9:10	Opening
9:10 - 9:50	Alexandr Kostochka: Minors in graphs with high chromatic number
Coffee Break	
10:10 - 10:30	S. Arumugam: On Dominator Colorings in Graphs
10:30 - 10:50	Eric Westlund: Hall's condition and partial proper m-colorings
10:50 - 11:10	Debbie Seacrest: The Gold Grabbing Game
Break	
11:20 - 11:40	Leanne Rylands: Results and conjectures on $\alpha$ -labellings of trees
11:40 - 12:00	Tao-Ming Wang: On Deficiency of Edge-Antimagic Graphs
12:00 - 12:20	Akito Oshima: Some results on partitional and other related graphs
Lunch Break	
1:50 - 2:10	Tyler Seacrest: Connections between Domatic and Chromatic Numbers
2:10 - 2:30	Heather Jordon: Alspach's Problem: The Case of Hamilton Cycles and
	5-Cycles
2:30 - 2:50	Elliot Krop: Nonmedian Direct Products of Graphs with Loops
2:50 - 3:10	Melanie Laffin: A survey on $(t, r)$ -regularity
Coffee Break	
3:30 - 3:50	Ian Roberts: k-regular antichains
3:50 - 4:10	Reza Akhtar: The Linear Chromatic Number of a Sperner Family
4:10 - 4:30	Younjin Kim: Large $B_d$ - free subfamilies
4:30 - 4:50	Rinovia Simanjuntak: On The Metric Dimension of Composition
	Product Graphs
Break	
5:00 - 5:20	Jerad DeVries: Edge-isoperimetric problems on products of regular
	graphs of an odd order
5:20 - 5:35	Steve Rosenberg: 1-Swappable Unicyclic Graphs
5:35 - 5:50	Sergei Bezrukov: New vertex-isoperimetric graph families
5:50 - 6:05	Uwe Leck: Maximal flat antichains of minimum weight
6:05 - 6:20	P. Selvaraju: Decomposition properties on harmonious labelings of some
	graphs

7:00 pm: Dinner at "The Shack", 3301 Belknap Street

### **Plenary Lecture**

### Alexandr Kostochka

9:10 am

Minors in graphs with high chromatic number

A graph H is a *minor* of a graph G if H can be obtained from G by deleting some edges and vertices and contracting some edges. In this case, we also say that G has an H-minor. Famous Hadwiger's Conjecture states that every graph with chromatic number k has the complete graph on k vertices as a minor. The conjecture is trivial for  $k \leq 3$ , easy to prove for k = 4, equivalent to the Four Color Theorem for k = 5, 6, and open for  $k \geq 7$ .

The aim of the talk is to survey recent progress in some problems closely related to Hadwiger's Conjecture.

### S. Arumugam, Jay Bagga, and K. Raja Chandrasekar 10:10 am On Dominator Colorings in Graphs

A dominator coloring of a graph G is a proper coloring of G in which every vertex dominates every vertex of at least one color class. The minimum number of colors required for a dominator coloring of G is called the dominator chromatic number of G and is denoted by  $\chi_d(G)$ . In this paper we present several results on graphs with  $\chi_d(G) = \chi(G)$  and  $\chi_d(G) = \gamma(G)$  where  $\chi(G)$  and  $\gamma(G)$  denote respectively the chromatic number and the domination number of a graph G. We also prove that if  $\mu(G)$  is the mycielskian of G, then  $\chi_d(\mu(G)) = \chi_d(G) + 1$  or  $\chi_d(G) + 2$ . We also present some results on the edge version of dominator colorings.

#### Sibel Ozkan and Erik Westlund

10:30 am

Hall's condition and partial proper m-colorings

A partial proper *m*-coloring of a graph *G* is a proper coloring  $\varphi : V_0 \to \{1, \ldots, m\}$ , for some  $V_0 \subseteq V(G)$ . Define the list-assignment  $L = L_{\varphi}$  by  $L(v) = \{\varphi(v)\}$  if  $v \in V_0$ , and  $L(v) = \{1, \ldots, m\} \setminus \{\varphi(N_G(v) \cap V_0)\}$  if  $v \in V \setminus V_0$ , where  $N_G(v)$  denotes the neighborhood of v.  $\varphi$  has a completion to a proper *m*-coloring of *G* if and only if *G* has a proper  $L_{\varphi}$ -coloring. We say (G, L) satisfies Hall's condition if, for all subgraphs H of G,  $|V(H)| \leq \sum_{\sigma \in \mathcal{C}} \alpha(H(\sigma, L))$ , where  $\alpha(H(\sigma, L))$  is the independence number of the subgraph of *H* induced on the vertices having  $\sigma$  in their lists. Hall's condition is necessary for *G* to have a proper *L*-coloring. *G* is said to be Hall *m*-completable, for some  $m \geq \chi(G)$ , if ever partial proper *m*-coloring  $\varphi$ , such that  $(G, L_{\varphi})$  satisfies Hall's condition, has a completion. In this talk, we give a brief survey of results and discuss some open problems.

#### Debbie Seacrest

The Gold Grabbing Game

A path of even length contains various amounts of gold at each vertex. Alice and Bob take turns removing leaves from the path and adding the gold from that leaf to their score.

Proving that Alice can secure at least half of the gold is a nice exercise. We generalize this result from paths to trees, proving a recent conjecture of Piotr Micek and Bartosz Walczak.

This is joint work with Tyler Seacrest, University of Nebraska - Lincoln.

10:50 am

Gunnar Brinkmann, Simon Crevals, Hadrien Mélot, Leanne Rylands, and Eckhard Steffen

11:20 am

Results and conjectures on  $\alpha$ -labellings of trees

Let T = (V, E) be a tree with n = |V|. A *labelling* of T is a bijection f from V to  $\{0, 1, \ldots, n-1\}$ . The induced label on each edge  $\{x, y\}$  is |f(x) - f(y)|. If these edge labels are distinct, then the labelling is said to be *graceful*. This notion was introduced by A. Rosa in 1967. The well known Graceful Tree Conjecture states that every tree has a graceful labelling.

A graceful labelling is an  $\alpha$ -labelling if there exists an integer k such that for each edge  $\{x, y\}$  of T either  $f(x) \leq k < f(y)$  or  $f(y) \leq k < f(x)$ . There are examples of trees that do not have an  $\alpha$ -labelling.

Some new results about trees which do not have  $\alpha$ -labellings will be presented, as well as some conjectures based on extensive computations.

#### Tao-Ming Wang

11:40 am

On Deficiency of Edge-Antimagic Graphs

An (a, d)-edge-antimagic labeling of a finite simple undirected graph with p vertices and q edges is a bijection from the set of vertices to the set of integers  $\{1, 2, \ldots, p\}$  such that the induced edge labelings form an arithmetic progression  $a, a + d, a + 2d, \ldots, a + (q - 1)d$ , where the induced labeling over an edge uv is f(u) + f(v). The (a, d)-edge-antimagic deficiency of a graph is defined to be the least number of isolated vertices one must add to make the graph (a, d)-edge-antimagic. We determine the deficiency of the (a, d)-edge-antimagic labeling for complete bipartite graphs among others, as a consequence, a conjecture raised by R.M. Figueroa-Centeno et al. in 2006 is confirmed.

#### Rikio Ichishima and Akito Oshima

12:00 noon

Some results on partitional and other related graphs

The notion of partitional labelings was recently introduced in [1] by the authors as a tool for finding sequential, harmonious and felicitous labelings of various classes of bipartite graphs which are defined in terms of cartesian products. A sequential labeling f of a graph G is called a *partitional labeling* if f satisfies the following conditions:

- 1. G is bipartite with two partite sets X and Y of the same cardinality s such that  $f(x) \leq t + s 1$  for all  $x \in X$  and  $f(y) \geq t s$  for all  $y \in Y$ , and the number of edges is 2t + s.
- 2. There exists a positive integer m such that the induced edge labels are partitioned into three sets [m, m+t-1], [m+t, m+t+s-1] and [m+t+s, m+2t+s-1].
- 3. There exists an involution  $\pi$  which is an automorphism of G such that  $\pi$  exchanges X and Y.
- 4. The s edges  $x\pi(x) \in E(G)$  for all  $x \in X$  have as labels the integers [m+t, m+t+s-1].

In this talk, we present several results on partitional graphs. Also, we show that the relationship between partitional graphs and graphs having other kinds of labelings. Moreover, we propose some open problems.

- R. Ichishima and A. Oshima, On partitional labelings of graphs, *Math. Comput. Sci.*, 3 (1) (2010) 39-45.
- [2] R. Ichishima and A. Oshima, On partitional and other related graph, preprint.

#### Tyler Seacrest

1:50 pm

Connections between Domatic and Chromatic Numbers

Much like a proper coloring of a graph is a partition of the vertices into independent sets, a domatic partition of a graph is a partition of the vertices into dominating sets. The domatic number of a graph is the number of parts in the largest domatic partition.

Perhaps the most basic result about the domatic number of a graph is that it is at least two, provided the graph has no isolated vertices. We extend this result in a way that gives surprising connection between the domatic number and the chromatic number.

#### Heather Jordon

2:10 pm

Alspach's Problem: The Case of Hamilton Cycles and 5-Cycles

In this talk, we will discuss the solution to Alspach's problem in the case of Hamilton cycles and 5-cycles; that is, we show that for all odd integers  $n \ge 5$  and all nonnegative integers h and t with hn + 5t = n(n-1)/2, the complete graph  $K_n$ decomposes into h Hamilton cycles and t 5-cycles and for all even integers  $n \ge 6$ and all nonnegative integers h and t with hn + 5t = n(n-2)/2, the complete graph  $K_n$  decomposes into h Hamilton cycles, t 5-cycles, and a 1-factor. We also settle Alspach's problem in the case of Hamilton cycles and 4-cycles.

### Kristi Clark and Elliot Krop

2:30 pm

Nonmedian Direct Products of Graphs with Loops

A median graph is a connected graph in which, for every three vertices, there exists a unique vertex m lying on the geodesic between any two of the given vertices. We show that the only median graphs of the direct product  $G \times H$  are formed when  $G = P_k$ , for any integer  $k \geq 3$  and  $H = P_l$ , for any integer  $l \geq 2$ , with a loop at an end vertex, where the direct product is taken over all connected graphs G on at least three vertices or at least two vertices with at least one loop, and connected graphs H with at least one loop.

### Melanie Laffin and Sibel Ozkan

 $2{:}50~\mathrm{pm}$ 

A survey on (t, r)-regularity

A simple, undirected graph is (t, r)-regular if the cardinality of the neighborhood set of t independent vertices is r. We present previously known general results as well as (2, r)-regular graphs in addition to recent work on (3, r)-regular graphs.

### Matthias Böhm and **Ian Roberts** k-regular antichains

The talk will focus on recent work on k-regular antichains.

An antichain is a collection  $\mathcal{A}$  such that for any distinct  $A, B \in \mathcal{A}, A \not\subseteq B$ . A k-regular antichain on [m] is an antichain in which each element of [m] occurs exactly k times.

When does a k-regular antichain of size n exist on [m]?

3:30 pm

#### Reza Akhtar

3:50 pm

The Linear Chromatic Number of a Sperner Family

A Sperner family S on a set S is called *k*-linearly colorable if there is a function  $f : S \to [k]$  such that if f(x) = f(y), then the subsets  $\{A \in S : x \in A\}$  and  $\{A \in S : y \in A\}$  of  $2^S$  are comparable by inclusion; the *k*-linear chromatic number of S is the least k for which S is *k*-linearly colorable. We prove a formula for the number of Sperner families (on a given set) of linear chromatic number 2. We also give tight upper and lower bounds for the maximum size of a Sperner family of any given linear chromatic number, and show that the Sperner families of maximum linear chromatic number are by far the most numerous.

This is joint work with Max Forlini (Miami University).

#### Younjin Kim

#### 4:10 pm

Large  $B_d$ - free subfamilies

Let  $f(\mathcal{F}, \Gamma)$  denote the size of the largest subfamily of  $\mathcal{F}$  having property  $\Gamma$ ,  $f(\mathcal{F}, \Gamma) := \max\{|\mathcal{F}'| : \mathcal{F}' \subseteq \mathcal{F}, \mathcal{F}' \text{ has property } \Gamma\}$ . Let  $f(m, \Gamma) := \min\{f(\mathcal{F}, \Gamma) : |\mathcal{F}| = m\}$ . First, we consider the case when  $\Gamma$  is the property that there are no four distinct sets in  $\mathcal{F}$  satisfing  $F_1 \cup F_2 = F_3$ ,  $F_1 \cap F_2 = F_4$ . Such families are called  $B_2$ -free. In 1972 Erdős and Shelah conjectured that  $f(m, B_2$ -free) =  $\Theta(m^{2/3})$ . We prove that Erdős and Shelah's conjecture is true and establish some general lower and upper bounds on  $f(m, B_d$ -free), where  $B_d$  is the Boolean lattice of dimension d. This is a joint work with Zoltán Füredi, Janos Barat, Ida Kantor, and Balazs Patkos.

H. Assiyatun, E.T. Baskoro, A.N.M. Salman, S.W. Saputro, **Rinovia Simanjuntak**, D. Suprijanto, and S. Uttunggadewa On The Metric Dimension of Composition Product Graphs

 $4{:}30~\mathrm{pm}$ 

A set of vertices W resolves a graph G if every vertex is uniquely determined by its coordinate of distances to the vertices in W. The minimum cardinality of a resolving set of G is called the metric dimension of G. In this talk, we give the general bounds of the metric dimension of a composition product of any connected graph G and H. We also show that the bounds are tight.

#### Jerad DeVries

5:00 pm

Edge-isoperimetric problems on products of regular graphs of an odd order

We present a 1-parametric family of graphs admitting nested solutions in the edgeisoperimetric problem on their cartesian powers. The graphs are obtained from a clique by removing from it one or more 2- factors. This way we also construct some of previously studied graphs obtained from cliques of an even order by removing 1-factors. However, for cliques of an odd order the derived graphs were not studied before.

### Amy Hlavacek, Dalibor Froncek, and **Steve Rosenberg** 1-Swappable Unicyclic Graphs

5:20 pm

We define the swapping number of an arbitrary simple graph, which is related to the edge reconstruction problem, and involves a weakening of the concept of a graph automorphism. Namely, a graph is called k-swappable if every edge is contained in some set of k edges which may be replaced by k non-edges to obtain a graph isomorphic to the original. We classify all 1-swappable trees by using a result of Harary and Lauri on edge reconstruction, and we proceed using original methods to classify all 1-swappable unicyclic graphs.

#### Sergei Bezrukov

 $5{:}35~\mathrm{pm}$ 

New vertex-isoperimetric graph families

A new family of graphs is presented, whose all cartesian powers have nested solutions in the vertex-isoperimetric problem. The graphs are obtained from grigs by adding to them some edges. This family involves new isoperimetric orders.

Martin Grüttmüller, Thomas Kalinowski, Sven Hartmann, **Uwe Leck**, and Ian Roberts Maximal flat antichains of minimum weight

5:50 pm

Maximal flat antichains of minimum weight

We study maximal families  $\mathcal{A}$  of subsets of  $[n] = \{1, 2, \ldots, n\}$  such that  $\mathcal{A}$  contains only 2-sets and 3-sets and  $A \not\subseteq B$  for all  $\{A, B\} \subseteq \mathcal{A}$ , i.e.  $\mathcal{A}$  is an antichain. For any n, all such families  $\mathcal{A}$  of minimum size are determined. This is equivalent to finding all graphs G = (V, E) with |V| = n and with the property that every edge is contained in some triangle and such that |E| - |T| is a maximum, where Tdenotes the set of triangles in G. The largest possible value of |E| - |T| turns out to be  $\lfloor (n+1)^2/8 \rfloor$ . Furthermore, if all pairs and triples have weights  $w_2$  and  $w_3$ , respectively, the problem of minimizing the total weight  $w(\mathcal{A})$  of  $\mathcal{A}$  is considered. We show that  $\min w(\mathcal{A}) = (2w_2 + w_3)n^2/8 + o(n^2)$  for  $3/n \leq w_3/w_2 =: \lambda = \lambda(n) < 2$ . For  $\lambda \geq 2$  our problem is equivalent to the (6,3)-problem of Ruzsa and Szemerédi, and by a result of theirs it follows that  $\min w(\mathcal{A}) = w_2 n^2/2 + o(n^2)$ .

### $P. \ Selvaraju$

Decomposition properties on harmonious labelings of some graphs

6:05 pm

### Participants

Reza Akhtar, Miami University, reza@calico.mth.muohio.edu

S. Arumugam, Kalasalingam University (India) and University of Newcastle (Australia), s.arumugam.klu@gmail.com

Jay Bagga, Ball State University, jbagga@bsu.edu

Sergei Bezrukov, University of Wisconsin - Superior, sb@mcs.uwsuper.edu

Barbara Brandt, University of Minnesota Duluth, bbrandt@d.umn.edu

Sylwia Cichacz, AGH University of Science and Technology, Krakow (Poland), cichacz@agh.edu.pl

Jerad DeVries, University of Wisconsin - Superior, jdevries@uwsuper.edu

Dalibor Fronček, University of Minnesota Duluth, dalibor@d.umn.edu

Heather Jordon, Illinois State University, hjordon@ilstu.edu

Younjin Kim, University of Illinois at Urbana-Champaign, ykim36@illinois.edu

Alexandr Kostochka, University of Illinois at Urbana-Champaign, kostochk@math.uiuc.edu

Don Kreher, Michigan Technological University, kreher@mtu.edu

Elliot Krop, Clayton State University, ElliotKrop@mail.clayton.edu

Melanie Laffin, Michigan Technological University, melanie.laffin@gmail.com

Uwe Leck, University of Wisconsin - Superior, uleck@uwsuper.edu

Sin-Min Lee, San Jose State University, lee.sinmin350gmail.com

Mirka Miller, University of Newcastle (Australia), mirka.miller@newcastle.edu.au

Akito Oshima, Tokyo University of Science (Japan), akitoism@yahoo.co.jp

Sibel Ozkan, Michigan Technological University, sozkan@mtu.edu

Ian Roberts, Charles Darwin University (Australia), Ian.Roberts@cdu.edu.au

Alexander Rosa, McMaster University (Canada), rosa@mcmaster.ca

Steve Rosenberg, University of Wisconsin - Superior, srosenbe@uwsuper.edu

### Participants (continued)

Leanne Rylands, University of Western Sydney (Australia), l.rylands@uws.edu.au

Debbie Seacrest, University of Nebraska - Lincoln, debbie.seacrest@gmail.com

Tyler Seacrest, University of Nebraska - Lincoln, s-tseacre1@math.unl.edu

P. Selvaraju, Vel Tech (India), pselvar@yahoo.com

Natarajan Selvi, ADM College for Women (India), dr.selvinatarajan@ymail.com

Sudhakar Shetty, Alva's Institute of Engineering and Technology (India), drsshetty@yahoo.com

Rinovia Simanjuntak, Institut Teknologi Bandung (Indonesia), rino@math.itb.ac.id

Hsin-hao Su, Stonehill College, hsu@stonehill.edu

Walter Wallis, Southern Illinois University, wdwallis@siu.edu

Tao-Ming Wang, Tunghai University (Taiwan), wang@go.thu.edu.tw

Erik Westlund, University of Wisconsin - Marshfield/Wood County, erik.westlund@uwc.edu