

Explanation of Quasi-Fuchsian Limit Set Animations

These are animations of the limit sets of one-complex-parameter families of infinite-covolume Kleinian groups, most of which are quasi-Fuchsian, so that the limit sets are quasicircles (topological circles that are fractal-like deformations of geometric circles). All of these groups are isomorphic to some quotient of a genus-2 surface group

There is a region of the complex plane in which all of these have been proven to be quasi-Fuchsian (and isomorphic to a genus-2 surface group), and the first animation is the limit point set as the parameter runs around the boundary of this region. The region in question is roughly diamond-shaped, symmetric with respect to the lines $\Re(z) = 1/2$ $\Im(z) = 0$. Note that there are two points in the animation where the limit sets are true circles – these are the Fuchsian boundary points, where the parameter is real. The two most complicated points in this animation occur where the imaginary part of the parameter attains its maximum and minimum (and where the real part is $1/2$).

The second animation runs along the line $\Re(z) = 1/2$ from $z = 1/2$ (also a Fuchsian point) to slightly outside the feasible region, then back. The extremely complicated portions of this animation are probably not quasi-Fuchsian, but it's not entirely clear how to describe them. Eventually (beyond the point where the animation “turns around”) the limit sets appear to fill the complex plane, indicating that if these are surface groups that may be embedded in a Kleinian group of finite covolume, then they must be virtual fibers of the Kleinian group.