MATHS 335 Take-Home Exam #1, Question #4

Here is what I was looking for in question 4: from question 1, we have

$$\begin{aligned}
x_0(n) &= r_1 s_1 x_1(n-1) + r_2 s_2 x_2(n-1) + r_3 s_3 x_3(n-1) + r_4 s_4 x_4(n-1)\\
x_1(n) &= s_0 x_0(n-1)\\
x_2(n) &= s_1 x_1(n-1)\\
x_3(n) &= s_2 x_2(n-1)\\
x_4(n) &= s_3 x_3(n-1) + s_4 x_4(n-1)
\end{aligned}$$

First, show that the substitutions

$$\begin{aligned}
x_0(n) &= \frac{x_4(n+4) - s_4 x_4(n+3)}{s_0 s_1 s_2 s_3}\\
x_1(n) &= \frac{x_4(n+3) - s_4 x_4(n+2)}{s_1 s_2 s_3}\\
x_2(n) &= \frac{x_4(n+2) - s_4 x_4(n+1)}{s_2 s_3}\\
x_3(n) &= \frac{x_4(n+1) - s_4 x_4(n)}{s_3}
\end{aligned}$$

may be used to eliminate x_0, x_1, x_2 and x_3. To do this, simply substitute each of these in turn into the last 4 equations above to get

$$\begin{aligned}
\frac{x_4(n+3) - s_4 x_4(n+2)}{s_1 s_2 s_3} &= s_0 \frac{x_4(n+3) - s_4 x_4(n+2)}{s_0 s_1 s_2 s_3} = \frac{x_4(n+3) - s_4 x_4(n+2)}{s_1 s_2 s_3}\\
\frac{x_4(n+2) - s_4 x_4(n+1)}{s_2 s_3} &= s_1 \frac{x_4(n+2) - s_4 x_4(n+1)}{s_1 s_2 s_3} = \frac{x_4(n+2) - s_4 x_4(n+1)}{s_2 s_3}\\
\frac{x_4(n+1) - s_4 x_4(n)}{s_3} &= s_2 \frac{x_4(n+1) - s_4 x_4(n)}{s_2 s_3} = \frac{x_4(n+1) - s_4 x_4(n)}{s_3}\\
x_4(n) &= s_3 \frac{x_4(n) - s_4 x_4(n-1)}{s_3} + s_4 x_4(n-1) = x_4(n)
\end{aligned}$$

Since all these equations are true (both sides are trivially equal) these substitutions eliminate x_0 through x_3, and eliminate the last 4 equations. We are left now with only the first equation. Make all the substitutions in it, then clear denominators to get

$$\begin{aligned}
x_0(n) &= r_1 s_1 x_1(n-1) + r_2 s_2 x_2(n-1) + r_3 s_3 x_3(n-1) + r_4 s_4 x_4(n-1)\\
\frac{x_4(n+4) - s_4 x_4(n+3)}{s_0 s_1 s_2 s_3} &= r_1 s_1 \frac{x_4(n+2) - s_4 x_4(n+1)}{s_1 s_2 s_3}\\
&\quad + r_2 s_2 \frac{x_4(n+1) - s_4 x_4(n)}{s_2 s_3}\\
&\quad + r_3 s_3 \frac{x_4(n) - s_4 x_4(n-1)}{s_3} + r_4 s_4 x_4(n-1)\\
x_4(n+4) - s_4 x_4(n+3) &= r_1 s_0 s_1 (x_4(n+2) - s_4 x_4(n+1))\\
&\quad + r_2 s_0 s_1 s_2 (x_4(n+1) - s_4 x_4(n))\\
&\quad + r_3 s_0 s_1 s_2 s_3 (x_4(n) - s_4 x_4(n-1))\\
&\quad + r_4 s_0 s_1 s_2 s_3 s_4 x_4(n-1)
\end{aligned}$$

Now, collect all the terms by index, and put $x_4(n+4)$ on the left-hand-side by itself to get

$$\begin{aligned} x_4(n+4) &= s_4 x_4(n+3) \\ &+ r_1 s_0 s_1 x_4(n+2) \\ &+ (r_2 s_0 s_1 s_2 - r_1 s_0 s_1 s_4) x_4(n+1) \\ &+ (r_3 s_0 s_1 s_2 s_3 - r_2 s_0 s_1 s_2 s_4) x_4(n) \\ &+ (r_4 s_0 s_1 s_2 s_3 s_4 - r_3 s_0 s_1 s_2 s_3 s_4) x_4(n-1) \end{aligned}$$

Re-index and simplify a bit to get

$$\begin{aligned} x_4(n) &= s_4 x_4(n-1) \\ &+ r_1 s_0 s_1 x_4(n-2) \\ &+ s_0 s_1 (r_2 s_2 - r_1 s_4) x_4(n-3) \\ &+ s_0 s_1 s_2 (r_3 s_3 - r_2 s_4) x_4(n-4) \\ &+ s_0 s_1 s_2 s_3 (r_4 s_4 - r_3 s_4) x_4(n-5) \end{aligned}$$

The last bit is simply an observation to get us from a recurrence relation involving $x_4(n)$ to the **same** recurrence relation involving $x(n)$. There's nothing for you to do here.