1.4-1. Let $ABC$ be a triangle, $G$ its centroid, and suppose that $BB'$ is congruent to $CC'$, where $B'$ is the midpoint of $AC$ and $C'$ is the midpoint of $AB$. Show that $GB$ is congruent to $GC$, and thus that $GBC$ is isosceles. Use that to show that triangle $BB'C$ is congruent to $CC'B$ and then conclude that $ABC$ is isosceles.

1.4-2. Let $ABC$ be a triangle, $G$ its centroid, and $A', B'$ and $C'$ be the midpoints of $BC$, $CA$, and $AB$, respectively. Recall that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. Use this to show that

$$\frac{2}{3}BB' + \frac{2}{3}CC' > BC$$

Add three inequalities like this to show that the sum of the medians is greater than $3/4$ of the perimeter.

For the other direction, complete the parallelogram $CABK$ and observe that twice the median from $A$ is $AK < AC + CK = b + c$. Add three inequalities like this to show that the sum of the medians is less than the perimeter.

1.5-1. Let $\ell$ be a line through $O$, and let $P_\ell$ be the image of $P$ under reflection through $\ell$. What this question is asking is to describe the shape of the set of all the $P_\ell$ as $\ell$ varies. Note that the distance from $P$ to $O$ is always the same as the distance from $P_\ell$ to $O$.

1.5-2. Multiply both sides of the first equation by $\Delta$. For the second, consider the equations

$$r = \sqrt{\frac{\Delta}{s}}$$
$$r_a = \sqrt{\frac{\Delta}{s - a}}$$
$$r_b = \sqrt{\frac{\Delta}{s - b}}$$
$$r_c = \sqrt{\frac{\Delta}{s - c}}$$

Multiply these four equations together.

1.5-3. Let the distances from $A, B, C$ to the points of tangency of the incircle be denoted $t_a, t_b, t_c$, respectively. Observe that

$$t_b + t_c = a$$
$$t_a + t_c = b$$
$$t_a + t_b = c$$

Use these to solve for $t_a$ and show that it’s $s - a$. Take a similar approach for the three excircles.

1.5-4. Use the “angle at the circumference is half the angle at the center” theorem.

1.5-5. You shouldn’t need a hint on this one...

1.6-1. This is the “easy” proof of concurrence of altitudes. It’s hard to give a better hint than the description in the problem.
1.6-4. Recall that the area of a triangle is half the product of any side with the altitude on that side. If two of these altitudes are equal, what does that say about the sides?

1.6-6. The length of the altitude is $b \sin C$. Now show that $b = 2R \sin B$.

1.6-7. Construct lines parallel to $BC$ through $G$ and $A$. Then, these lines and $BC$ must cut any transverse line proportionally. The median is cut in a $2:1$ ratio, so the altitude must also be cut in a $2:1$ ratio. What does this say about the perpendicular distance from $G$ to $BC$?

1.6-8. Suppose the Euler line of triangle $ABC$ passes through $A$. If the angle at $A$ is not a right angle, show that $AH$ ($H$ is the orthocenter) must be a median (that is, it must also pass through the midpoint of $BC$) in addition to being an altitude.