Learner Centered Pedagogy in a Liberal Arts Mathematics Course

Abstract: We discuss the pedagogical redesign of a liberal arts mathematics course, which enrolls roughly 3000 students per year. Flipping the classroom is a prominent feature of the new pedagogical approach. We describe the nature of this redesign, course resources, and how students are assessed for both preparation and understanding. We also indicate how the course redesign was implemented, and we assess its success. The results thus far are encouraging. For example, an analysis of course grades indicates increased performance for students participating in the redesigned course, but we are cognizant of several confounding factors.

Key Words: flipped classroom, liberal arts mathematics, learner-centered teaching

1. INTRODUCTION

The purpose of this article is to describe how flipped instruction is being introduced, on a large scale, in a liberal arts core mathematics course. We describe the development of course materials (activities and resources), implementation, assessment, and the success of the redesigned pedagogy. In this introductory section, we provide information about the course, motives for introducing flipped instruction, and the structure of this article.

These efforts are taking place at a large state university enrolling approximately 17,000 students. All undergraduate students must take a mathematics course to fulfill a core curriculum requirement. While there are several mathematics courses that satisfy this requirement, most students, especially those who are not majoring in STEM fields, enroll in a course titled Mathematics and Its Applications, hereafter referred to as MAP. The course catalog description of MAP is as follows:

A diverse course including statistics and other topics such as mathematical modeling, problem solving, finance, geometrical concepts, growth patterns, and applications to the physical sciences, social sciences, and economics.

Recommended background: three years of college preparatory mathematics in high school.
When measured in terms of grades, student success rates in MAP have historically been fairly good. Nearly 80% of the students who enroll in MAP in a given term earn a C− or better. However, given that MAP is the last collegiate mathematics course many students take, and that on-time graduation plays such a significant role in state funding metrics for public universities, there is significant interest on the part of both faculty and university administration in improving student performance in MAP.

Efforts to improve performance in MAP focused on increasing student engagement with higher-level course material, peers, and faculty. To accomplish this, flipping and associated resources were introduced into the course. Briefly, flipped (or inverted) instruction occurs when the majority of information transfer, and routine checks of understanding, occur outside of class, while class time is primarily devoted to engaging students in higher-level tasks and collaborative work under the guidance of the instructor. Recent articles describing flipped instruction in mathematics classrooms include [9] and [10]. As Talbert points out in [9], students in the flipped classroom model have the advantage of engaging in tasks high on Bloom’s Taxonomy [2] (applying, analyzing, evaluating, creating) under the guidance of an instructor.

Flipped instruction, along with other high-engagement pedagogies such as the Moore Method [4], is a means by which one can engage in learner-centered instruction, as envisioned by Weimer [11]. (See [6] for a thoughtful review of [11].) Learner-centered teaching is characterized by five principles, listed here as in [11]:

1. It is teaching that engages students in the hard, messy work of learning.
2. It is teaching that motivates and empowers students by giving them some control over learning processes.
3. It is teaching that encourages collaboration, acknowledging the classroom (be it virtual or real) as a community where everyone shares the learning agenda.
4. It is teaching that promotes students’ reflection about what they are learning and how they are learning it.
5. It is teaching that includes explicit learning skills instruction.

Evidence for the efficacy of learner-centered teaching in mathematics and physics is presented, for example, in [1] and [8], respectively.
Operating on an internal grant, five faculty members spent Summer 2013 developing and gathering resources to support this new flipped pedagogical approach. Resources include the following items, which are discussed later in greater detail:

- Organizational materials such as a sample syllabus, class calendar, and suggested homework assignments.
- A commercial product called Vizi consisting of short video modules and exercises. This is intended for use outside of class. (Vizi was already in place prior to this project, but perhaps was under-utilized.)
- A document cross-referencing text sections with Vizi lessons and master syllabus objectives.
- Presentation slides and accompanying voice-over, intended for use outside of class.
- Early assessment activities to measure student engagement, to be conducted outside of class.
- In-class activities and group projects.
- Blackboard course site housing materials contributed by, and shared by, all MAP instructors.

The structure of this paper is as follows: A general redesign overview is provided in Section 2. Resources to aid students in their preparation for class are described in Section 3, while in-class activities and group projects are discussed in Section 4. Student assessment is discussed in Section 5. Implementation and success of the MAP redesign are addressed in Section 6.

2. GENERAL REDESIGN OVERVIEW

Instructors pick and choose from the materials listed in the table. Each item listed will be discussed later in the paper.

<table>
<thead>
<tr>
<th>Resources for student preparation before class</th>
<th>Resources for use in class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Online interactive Vizi lessons</td>
<td>“Mini-lecture” templates</td>
</tr>
<tr>
<td>The electronic textbook</td>
<td>Quick concept checks</td>
</tr>
<tr>
<td>Presentation slides with accompanying videos</td>
<td>Activities and Projects</td>
</tr>
<tr>
<td>“Finish the Problem” quizzes</td>
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<tr>
<td>Early assessment quizzes</td>
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It is up to individual instructors to determine how these components weigh in the course grade, but the sample syllabus provided with the materials recommends that 15% of a student’s grade be determined by the preparatory work done outside of class. Furthermore, it suggests that an additional 25% of a student’s grade come from in-class projects and assignments and that the remaining 60% be tied to written tests.

3. RESOURCES FOR STUDENTS OUTSIDE OF CLASS

Establishing materials for student use outside of class is essential for a flipped classroom. The types of these materials we developed for MAP are described below.

3.1 Vizi Courseware

Vizi Courseware is an online, interactive learning system equipped with closed captioning that is a required resource for all students taking MAP. There are Vizi products in nine disciplines, which are marketed across the country. We are using the Vizi product titled “Math in the Liberal Arts,” which was authored by a member of our faculty who is very experienced in teaching MAP. In Vizi, the course material is divided into eight lessons, with each lesson divided into six to eight segments. A single lesson in Vizi corresponds to a chapter (or more) of material in a textbook and covers a particular topic such as probability or statistics. The introduction of each lesson includes a rationale for the inclusion of the material in that lesson. In most segments of each lesson, students are first presented with a concept either through a brief lecture or demonstration. Then students are guided through interactive examples where they select an answer or type in an answer to a routine question and are given immediate feedback and prompts to more information as needed. Students subsequently work independently through additional problems, also with immediate feedback. The last segments of each lesson feature interactive vocabulary flashcards, a quick review quiz with immediate feedback, and an assessment which is sent to the instructor for grading. An outline of notes is available for each Vizi lesson which instructors can make available to students through Blackboard, our online course management system. Vizi itself is also accessed through Blackboard. Typically students are assigned a segment or two in Vizi to work through before a particular class session. Students are encouraged to complete the notes while viewing the lesson. This keeps students engaged while watching the videos and provides them with a complete set of notes on the new material.
3.2 The Electronic Textbook

One common way to ask students to be prepared for class is to have them read specific sections of a textbook. For our MAP classes, we adopted the electronic version of the text *Quantitative Literacy: Thinking Between the Lines* by Crauder, Evans, Johnson, and Noell [5]. Students pay for this and Vizi as a part of their registration fees. One advantage of this process is that all students have access to these materials on the first day of class. There is no waiting for financial aid or for students to go to the bookstore.

As with many textbooks, this textbook offers routine “Try it Yourself” problems in each section. The answers to each of these exercises are included. This is a great opportunity for students to check their understanding. If the student is unable to get the correct answer, an example in the text immediately preceding the problem offers a similar problem with a step-by-step solution. Several instructors ask students to work through these problems as part of their preparation for class. Also, some instructors ask students to create notes (outlines) from the text which include important definitions and key concepts.

The electronic textbook is accessed through Blackboard. Students can highlight, add notes, and even send a note to the instructor if a particular example or topic is confusing. Students can also form study groups and have interactive conversations about the assigned reading. Furthermore, notes and supplemental materials (PDFs, links to videos, etc.) can be posted by the instructor for all of the students to see. We have not yet fully explored or documented the use of these features of the electronic textbook.

3.3 Presentation Slides with Accompanying Videos

We created presentation slides for each chapter that contain definitions and worked examples. These slides contain minimal duplication of the content in Vizi. When paired with the related Vizi assignment, the students receive the basic background information needed to complete most of the suggested textbook assignment or prepare for a classroom activity. We also used a desktop recorder to create videos from these slides so that we could record verbal explanations. The idea was to appeal to both visual and auditory learners.

4. IN-CLASS RESOURCES
The idea of flipping the classroom is new to many of our instructors, so we provide them with many resources for in-class use. We discuss ways of dividing students into groups, and we encourage instructors to regularly visit each group during class to make sure they stay on task and see if they have questions. When a student asks a question, we encourage instructors to involve the student’s group or nearby groups to help them over the hurdle. Examples of in-class resources are given below. Instructors choose the resources which best fit their personal style.

### 4.1 “Mini-Lecture” Templates

For several sections of the text, we created short fill-in-the-blank notes that contain relevant definitions and a few introductory examples. Instructors can then spend a brief portion of class time giving students some of the main ideas they need to get started on their work. The time that is spent working on these as a class gives students an opportunity to ask questions or reinforce what they have done in their preparation, and is a quick opening activity. Furthermore, many students complete their preparatory work in advance and typically appreciate a quick reminder before beginning their day’s work.

### 4.2 Quick Concept Checks

We provide several short classroom exercises which can be used as quick checks for understanding. These concept checks can be collected for attendance or self-graded. There are some areas where a particular concept or idea must be mastered before students can be successful in the next topic. For example, a student needs to be able to calculate a monthly payment to be able to actively engage in an upcoming lesson involving finding the best deal on several car loan options. Another one of our quick checks involves calculating the standard deviation by hand. Although the use of technology is allowed and encouraged, many feel that seeing how this value is calculated may help students better understand what the statistic represents, as well as how this value changes with data sets. These activities can be done together as a class, in small groups, or individually.

### 4.3 Activities and Projects

Some of the in-class activities require students to take an in-depth look at real life situations. For example, one of the goals of this course is to enable our students to become wise consumers of statistics. Students are encouraged to ask questions about survey results and graphs that they see in newspapers, magazines, and online. What
conclusions can be drawn from the data? How are the questions worded? Are there any sources of bias? Several activities are designed to help students perform this analysis. One such activity asks students to analyze examples of data presentations that are found online. In this activity, students examine sample size and margins of error. Is the margin of error acceptable for the data presented? Is the graph misleading in any way? Typically, instructors lead the entire class in completing questions related to a given article and graph, and then students work in groups on a second article and graph. (This activity may be viewed in Appendix A.) Additional activities require students to bring in examples of statistics that they have found on their own or involve the class in generating data to form a normal distribution.

Another in-class activity demonstrates voting power. In this activity, the class is randomly divided into groups. Each group is given a short set of questions on a current social issue, such as immigration or education. Each group then predicts how voters might respond to these questions based on party affiliation, gender, race, ethnicity, and geographical location. After this has been completed, each student is assigned a set of demographics for a fictional voter. The students in each group then determine how much power their voter would have if they based their vote along party lines or gender or another of the demographic groupings. Through this activity students can learn how power can shift. They can also learn how to maximize their voting power through forming coalitions wisely.

5. STUDENT ASSESSMENT

No course is complete without a plan for student assessment. There must be a system for measuring students’ preparation for class, as well as a system for determining the level of student understanding and achievement. A key consideration is the prompt identification of students who may be straying from a path for success. We developed assessment instruments for this purpose that provide immediate feedback to both the instructor and the student. Several student assessment items are discussed below.

5.1 “Finish the Problem” Quizzes

In order for the flipped classroom to be effective, there must be some way to ensure that students do the necessary background work. Some authors have utilized what they call “tickets” for entry to class (see [3] and [5]). In other
disciplines, an outline or idea map based on an assigned reading might serve this purpose. We have created short “finish the problem” videos using a tablet application. (An example may be viewed at https://www.educreations.com/lesson/view/5-1-finish-the-problems-125/22537055/?s=zB0dL0&ref=link.) These short videos illustrate the solution for a sample problem, and then offer hints to get students started on a similar problem. Students are then required to bring their completed solution to the next class. The first few minutes of class can be devoted to a student or instructor presentation of the solution and class discussion. Instructors can decide how to allot points.

5.2 Early Assessments

We provide very short (two to four questions) “quizzes” that are produced, accessed, administered, and graded on Blackboard. Some instructors allow students multiple attempts. These are typically not worth very many points and contain simple questions that students can answer easily if they have done the reading. Instructors can determine at a glance which students are doing the necessary outside-of-class work.

5.3 Vizi Assessments

Students can be assessed on the material presented through Vizi in a variety of ways. Some faculty members require students to keep all of their course materials in a folder that is collected and graded periodically. Included in this material are the completed Vizi notes and any written work from the Vizi examples and problems. Other instructors use the completed notes as a ticket to get into class the day after the assignment has been given. The final assessment in each Vizi lesson, if assigned, is sent directly to instructors to use as they see fit.

5.4 In-Class Exams

Most instructors are still using traditional formal assessments. One advantage of offering traditional formal assessments is that they allow the new instruction to be compared to more traditional classrooms. To better facilitate this comparison, we plan to add several common exam questions for each chapter.

6. IMPLEMENTATION AND ASSESSMENT OF THE REDESIGN
In this section, we describe the implementation of the learner-centered approach, assess the success of the redesign, and indicate areas for improvement.

6.1 Implementation:
An internal grant provided a substantial tailwind to the development and implementation of the MAP pedagogical redesign. The grant, totaling $29,500 over 2013–2015, provided for the following items:

- Summer 2013 stipends for five faculty members (the designers) to create new materials and pedagogical strategies for MAP.
- Workshops conducted by the designers to introduce the new materials and the new pedagogical strategy to faculty and graduate students.
- Stipends for faculty to pilot the new materials/strategy extensively (approximately six each semester from Fall 2013 through Spring 2015).
- Summer 2014 stipends for two faculty members to assess the effectiveness of the redesign and to suggest improvements.

Implementation began in Summer 2013 with two workshops introducing the new materials and pedagogical approach to faculty and graduate assistants. The designers gave an overview of the Blackboard site containing the resources and offered helpful suggestions regarding their use. There was also time for questions. The sessions were well attended by both faculty and teaching graduate assistants, and indicated far more immediate faculty buy-in than was originally expected.

Open conversation sessions were scheduled throughout the academic year for instructors to meet and informally discuss their thoughts on the resources. Many made constructive suggestions or provided activities to add to the resources. Generally, faculty needed time to adjust to the new pedagogy, just as students did.

From Fall 2013 through Spring 2015, approximately six faculty members per semester received stipends to pilot the new materials and pedagogy, but from the start between two-thirds and three-quarters of MAP instructors (out of approximately 25 MAP instructors total) have chosen to use the new materials and pedagogy regularly.
Further workshops were conducted to prepare faculty and graduate student teaching assistants to use the materials in Fall 2014 and Fall 2015. Also, an additional website has been created on Blackboard with a streamlined version of the resources. It features a syllabus, calendar and assignments, and the activities needed to completely outfit a class. This was designed for teaching graduate assistants and first-time instructors so that they would not have to decide which activities (and how many) to pull for a single semester.

6.2 Assessment of Student Performance:

Our analysis of student performance in MAP rests upon final course grades from Fall 2012 through Spring 2015, excluding summer courses and online courses. Beginning in Fall 2013, all MAP instructors were given a survey (Appendix B) that sought information about how they were using the new MAP materials/pedagogy. Based on the results of these surveys, each MAP instructor was classified as learner-centered (new materials being used at least weekly) or traditional. It is important to note that:

- All instructors teaching MAP in Fall 2012 or Spring 2013 were classified as traditional. (Recall that this was prior to the development and implementation of the new materials/pedagogy.)
- An instructor is classified as both traditional and learner-centered if he/she used both methods of instruction sometime between Fall 2012 and Spring 2015 (e.g., traditional in Fall 2013 and learner-centered in Spring 2015). This instructor’s name would appear on the list of traditional instructors and on the list of learner-centered instructors. (Later we will be interested in this set of instructors.)

For each traditional instructor, the percentage of each type of letter grade (A,B,C,D,F,W) was computed over all of the traditional MAP course sections taught by that instructor during the three-year period. (Regardless of how many sections are taught, each instructor is assigned a single six-tuple of grade percentages.) An analogous computation was made for each learner-centered instructor.

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1 We chose not to separate fall semesters from spring semesters, regardless of the common belief that fall grades differ significantly from spring grades. A chi-square test of independence applied to our fall and spring grades (p-value 0.45) does not suggest that fall grade distributions differ significantly from spring grade distributions.

2 We chose not to organize our data by MAP course section because some instructors teach many more MAP sections than others. Organizing our data by instructor, rather than course section, ensures that each instructor’s MAP grade distribution carries equal weight in our analysis.

3 It should be pointed out that MAP underwent a textbook change in Fall 2013. A chi-square test of independence for grade distributions suggests that we should not conclude that the change of textbook resulted in a change of grade distribution.
The graphic below and corresponding table (Figures 1 and 2, respectively) illustrate mean percentages of each letter grade, with the mean taken over learner-centered instructors and over traditional instructors.

![Mean Percentages of Letter Grades](image)

**Figure 1**

<table>
<thead>
<tr>
<th>Mean letter grade percentages</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>F</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall 2012 through Spring 2015</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traditional</td>
<td>20.4</td>
<td>29.7</td>
<td>23.4</td>
<td>8.7</td>
<td>8.0</td>
<td>6.9</td>
</tr>
<tr>
<td>Learner-Centered</td>
<td>25.3</td>
<td>31.5</td>
<td>21.0</td>
<td>6.8</td>
<td>7.1</td>
<td>8.3</td>
</tr>
</tbody>
</table>

**Figure 2**

A glance at these data summaries suggests that:

- learner-centered instructors assign a smaller percentage of C's and D's than traditional instructors
- learner-centered instructors assign a larger percentage of A's and B's than traditional instructors
- there might be little difference in combined percentage of F's and W's between learner-centered instructors and traditional instructors.

It is important to determine whether these thumbnail observations carry statistical significance. Some statistically significant results (one-tailed t-tests at the 0.10 level) are given in Figure 3 below. In that figure, the heading "Sample Size" counts instructors, with "L" denoting learner-centered and "T" denoting traditional, and “Significant results” gives results of the form (±Grade, p-value) where the "+" indicates more for learner-centered than for traditional, and "-" indicates less for learner centered than for traditional. The p-values are those for one-tailed t-tests.
tests. Finally, concatenated grades denote sums of percentages of individual grade types (e.g., CD's denotes the sum of the percentage of C's and the percentage of D's).

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Significant Results: one-tailed t-test, 0.10 level</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L, T)</td>
<td>(+A, 0.062) (+AB, 0.047) (-C, 0.087) (-D, 0.075) (-CD, 0.047)</td>
</tr>
</tbody>
</table>

These results support our observations.

Recall that some instructors are classified both as traditional and learner-centered. We can, at the expense of sample size, control for instructor variation by analyzing only this set of instructors. With this restriction in place, we have the following data summary and list of statistically significant events (Figures 4, 5, and 6):

![Mean Percentage of Letter Grades Fall 2012 through Spring 2015](image)

**Figure 4**

<table>
<thead>
<tr>
<th>Mean letter grade percentages</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>F</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall 2012 through Spring 2015</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traditional</td>
<td>17.4</td>
<td>28.8</td>
<td>25.6</td>
<td>12.5</td>
<td>8.8</td>
<td>5.3</td>
</tr>
<tr>
<td>Learner-Centered</td>
<td>25.0</td>
<td>28.5</td>
<td>22.6</td>
<td>5.9</td>
<td>9.1</td>
<td>8.9</td>
</tr>
</tbody>
</table>

**Figure 5**
<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Significant Results: one-tailed $t$-test, 0.10 level</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L, T)</td>
<td>($\pm$Grade, p-value)</td>
</tr>
<tr>
<td>(13, 13)</td>
<td>(-D, &lt;0.01) (-CD, 0.026) (+W, 0.047) Nearly significant: (+A, 0.138)</td>
</tr>
</tbody>
</table>

Restricting to the set of instructors who taught using both methods does not support our observations as strongly as we would like: there is significant increase in percentage of W's for learner-centered instructors, and the increase in percentage of A's for learner-centered instructors is not quite significant. However, our sample size is quite small.

7. CONCLUSION AND FUTURE RESEARCH

Our work shows that it is possible to bring the benefits of learner-centered pedagogy to high-enrollment courses—even courses taught by a diverse cadre of full-time, part-time, and graduate student instructors. In our setting, ongoing leadership from a core group of faculty members, support from university administration, and faculty buy-in were important factors in this project.

While we are encouraged by our progress, there is still much for us to learn about how this project is affecting MAP student performance, student attitudes, and instructor attitudes. Regarding assessment of student performance, course-wide student assessments (common final exam questions and the like) should be used to study student performance in addition to course grades.

Unfortunately, the faculty survey is void of attitudinal questions concerning the redesign, and this is clearly an area that needs more research. Faculty opinions were informally gathered during conversation hours throughout the years, and these sometimes prompted important changes in the materials. For example, some instructors wanted an early assessment tool that incorporated content that required a deeper understanding of the material than these quizzes did. During Summer 2014, “Finish the Problem” quizzes were added for many of the chapters. (See Section 5.1.) Throughout, we have made additions and changes to the Blackboard resource site in response to constructive criticism, but a more systematic study of faculty opinions needs to be undertaken.
Many of the piloting instructors administered anonymous student surveys (see Appendix C). These surveys sought student opinion on various resources. The student polls suggest that many students are utilizing the resources given to them outside of class, and that those who are using the resources generally find them to be helpful. As expected, quotes from students indicate that there are mixed feelings about the flipped classroom pedagogy. This approach requires students to complete more preparation before coming to class, and it may seem unnatural to them at first after years of learning in a lecture-based environment. In all, more work needs to be done in gathering and analyzing student attitudinal data.

REFERENCES


